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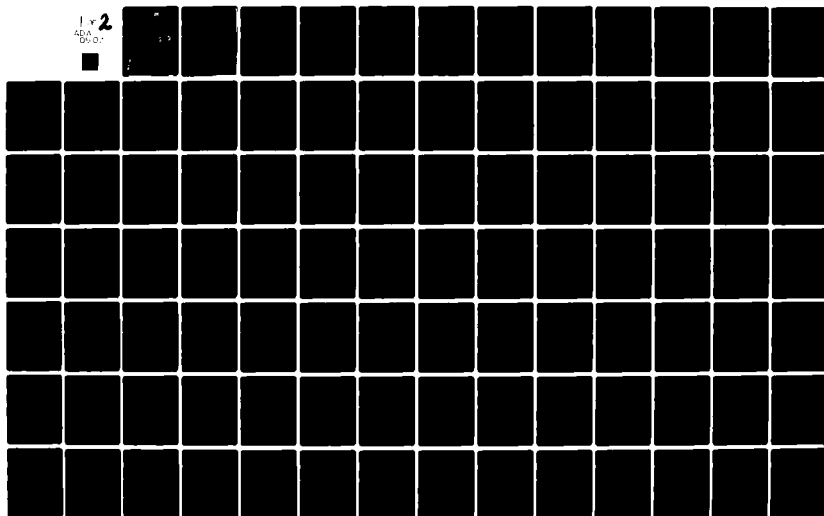
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MILITARY GEODESY AND GEOSPACE SCIENCE  
Unit Two

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The topics covered herein are intended to provide conceptual rather than working knowledge. Ideally, the student completing this course will have attained a broad understanding of the MC&G field and will be able to develop specialized expertise quickly when required.

The notes are intended to be presented in chapter/section order within each of the four Units of Instruction. However, several of the subsections in these notes contain more advanced material which may be omitted without loss of continuity. These subsections are denoted with the symbol (†) after the title. A fifth volume contains faculty material.

The organizational flow of the lectures is from concepts in the initial sections, particularly in Unit One, to applications and specific systems later on. As a result the student is often referred ahead to provide motivation in regard to relevancy. In later chapters, however, the situation is reversed with the student referred back to review important conceptual material as necessary.

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## FOREWORD

This lecture course provides a full-year introduction to Military Geodesy and Geospace Science. Throughout the presentation a military perspective is maintained which links Mapping, Charting, and Geodesy (MC&G) issues with modern defense requirements. Elementary preparation is assumed in the subjects of general physics, mechanics, chemistry, astronautics, and linear system theory. The student should also be familiar with differential equations, analytic geometry, and linear algebra. Some acquaintance with vector calculus is useful but not essential.

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GLOSSARY OF ACRONYMS AND  
ABBREVIATIONS FOR UNIT TWO

ADX	- Area Depth Transformation	2-88
ALCM	- Air Launched Cruise Missile	2-75
ALS	- Azimuth Laying Set	2-47
APPS	- Analytical Photogrammetric Positioning System	2-115
BATH	- Best Available True Heading	2-46
BC-4	- A precision telescopic camera used for satellite tracking	2-10
CEP	- Circular Error Probable (the radius defining the circle about a target, inside of which fifty percent of a weapon system's warheads will impact)	2-3
CONUS	- Continental United States	2-74
DLMS	- Digital Land Mass System	2-117
DMA	- Defense Mapping Agency	2-114
DOD	- Department of Defense	2-6
DTM	- Digital Terrain Models	2-125
EGM	- Earth Gravity Model	2-36
G&G	- Geodetic and Geophysical	2-23
GDOP	- Geometric Dilution of Precision	2-104
GPS	- Global Positioning System	2-48
HIRAN	- High-Precision SHORAN	2-6
ICBMs	- Long-range intercontinental ballistic missiles	2-2
INS	- Inertial Navigation System	2-13
JOGs	- Joint Operations Graphics (a series of maps)	2-115

GLOSSARY OF ACRONYMS AND  
ABBREVIATIONS FOR UNIT TWO (Continued)

JTIDS	- Joint Tactical Information Distribution System (a radio positioning system)	2-106
LOPs	- Lines of position	2-104
LORAN	- <u>L</u> ong <u>R</u> ange <u>N</u> avigation (a radio navigation system)	2-48
Loran-C	- <u>L</u> ong <u>R</u> ange <u>N</u> avigation (C represents an upgraded version of the system)	2-105
LOS	- Line of sight	2-81
LRGM	- Launch Region Gravity Model	2-39
MAD	- Mean Absolute Difference	2-90
MARC	- Modified Azimuth Radar Correlator	2-133
MC&G	- Mapping, Charting and Geodesy	2-1
MIAD	- Minimum Integrated Absolute Difference	2-96
MISD	- Minimum Integrated Squared Difference	2-96
MX ICBM	- <u>M</u> issile <u>X</u> - A Long-Range Inter <u>C</u> ontinental <u>B</u> allistic <u>M</u> issile (currently under development)	2-75
NAVSTAR	- <u>N</u> avigation <u>S</u> ystem Using <u>T</u> ime And <u>R</u> ange (a satellite-based radio navigation system) -- also called GPS	2-110
NNSS	- Navy Navigational Satellite System	2-9
OMEGA	- Global radio navigation system	2-103
OUSD/R&E	- Office of the Under Secretary of Defense for Research and Engineering	2-114
PDMM	- Pulse Doppler Map-Matching	2-141
PPDB	- Point Positioning Data Bases	2-115
PLRS	- Position Location Reporting System (a radio positioning system)	2-106
RADAG	- Radar Aimpoint Guidance	2-116

GLOSSARY OF ACRONYMS AND  
ABBREVIATIONS FOR UNIT TWO (Continued)

RIV	- Relative Information Vector	2-135
ROCS	- Range Only Correlation System	2-133
RMS	- Root mean square	2-99
RTS	- Radar Terrain Sensor	2-88
SARSIM	- Synthetic Aperture Radar Simulation	2-131
SECOR	- Sequential Collation of Range (a radio positioning system)	2-9
SHORAN	- <u>Short Range Navigation</u> (which employs the time of travel of pulse-type transmission from two or more fixed stations to measure slant-range distance from the stations)	2-6
SLBM	- Sea Launched Ballistic Missile	2-75
TACAN	- Tactical Air Navigation (an omnidirectional military radio navigation system)	2-108
TERCOM	- Terrain Contour Matching	2-87
TOA	- Time of arrival	2-106
TRANET	- <u>Transit</u> (Navigation Satellite) <u>Network</u> (Navy)	2-12
TRANSIT	- Name of a navigation satellite (Navy Navigation Satellite also called NAVSAT)	2-109
UHF	- Ultra High Frequency (300 to 3000 MHz)	2-108
USGS	- United States Geological Survey	2-114
USN	- United States Navy	2-67
VOR	- Visual Omnirange (a radio navigation system which provides heading direction information)	2-103
VORTAC	- <u>Very</u> high frequency <u>Omnirange</u> <u>TACAN</u>	2-103
WGS60	- DoD World Geodetic System 1960	2-6
WGS66	- World Geodetic System 1966	2-7
WGS72	- World Geodetic System 1972	2-9

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## UNIT TWO

### WEAPON SYSTEMS AND MC&G

#### CHAPTER ONE INTRODUCTION

The general terrestrial mapping, charting, and geodesy (MC&G) problem can best be formulated as the problem of generating a metric<sup>\*</sup> description of the earth and its physical properties -- a description in which relative positions within and on the surface of the earth are accurately described, the figure of the earth is accurately portrayed, and the gravitational, magnetic, and spin properties of the earth are fully represented in quantitative terms. Broader and more comprehensive definitions of the MC&G problem might be found within the domain of the planetological<sup>\*\*</sup> sciences, but the concept of quantitative metric relationships resulting from actual measurement processes will always be central to the definition.

In the context of modern weapon systems, MC&G applications tend to concentrate in the areas of relative position determination on the surface of the earth and quantitative

---

\*That is, a description characterized by quantitative expressions of relationships.

\*\*Referring to an extension of the concepts of geology and geophysics to include the other planets as well.

characterization of the terrestrial gravity field.<sup>\*</sup> The classical long-range artillery problem, for example, requires highly accurate knowledge of gun and target locations with respect to a common datum in a well-defined local coordinate system, together with approximate characterization of the gravity field, if a reliable one-shot hit capability is required. Practical constraints on achieving this capability, due partly to gun/target relative location limitations and partly to non-geodetic problems in the aiming process, lead to a multiple-shot, shoot-sense-correct philosophy that persists to this day.

The advent of high-cost, inertially guided short-to-medium-range ballistic missile systems of the Lance/Pershing/Jupiter/Thor class in the late 1950s and early 1960s, however, led to fundamental changes in this philosophy. In terms of mission design, these weapon systems may logically be viewed as the extrapolation of long-range artillery systems to modern, high-complexity, high-attrition rate, nuclear engagement scenarios. In such scenarios, force levels are limited by high unit costs, and weapon system response time is critical. The classical multiple-shot, shoot-sense-correct approach is no longer viable, and much greater emphasis is placed on successful single-shot kill capabilities. The nuclear payloads of these newer weapon systems tend to reduce the penalty for inaccurate delivery, but the requirement for a one-shot target neutralization capability correspondingly increases the need for accurate relative positioning of the launch point and the target.

Long-range intercontinental ballistic missiles (ICBMs) have introduced a fundamentally new problem into the weapon

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<sup>\*</sup>This scope of interest is likely to broaden considerably -- to include magnetic phenomena and detailed surface geophysical phenomena -- as future weapon systems requirements continue to emerge.

system targeting process: the determination of launch point and target point relative positions over intercontinental ranges. Up until this time, relative positioning could be accomplished by locating the launch point and the target point with respect to a common datum and a single land mass. Line-of-sight, direct survey techniques could be used<sup>\*</sup> to determine their relative positions. But the need for trans-oceanic surveys introduced a dependence on the metric representation of the entire figure of the earth to tie the continental datums together in a well-defined world-wide geodetic coordinate system.

For reasons<sup>\*\*</sup> that will be discussed later in this Unit, a need for a more accurate global representation of the earth's gravitational field also emerged at this time, and the higher-order spherical harmonic gravitational models (Section 3.2 of Unit One) derived from relatively recent satellite tracking data became of immediate interest. These models permit far more accurate predictions of gravitational contributions to missile trajectory dynamics than could be obtained from simpler figure-of-the-earth models, and allow gravitational modeling errors to remain a reasonably small fraction of the weapon system Circular Error Probable (CEP),<sup>‡</sup> even at intercontinental ranges.

Continual improvements in ballistic missile system technology have gradually reduced the inaccuracies in the inertial navigation system hardware and the endo-atmospheric

---

\*At least theoretically; other techniques were also commonly used.

\*\*Associated with the basic physics of inertial navigation systems; the net result is a need for a priori knowledge of the relationship between missile position and gravitational acceleration, to support the missile targeting process.

‡This is the radius of a circle centered at the target, within which 50 percent of the missiles launched against that target would fall, considering their probable miss distributions.

dispersions of the reentry systems (the two largest earlier contributors to weapon system CEP), so that by the mid-1960s, residual errors in the gravitational models threatened to constrain further accuracy improvement attempts. The problem was not misrepresentation of the long wavelength global characteristics of the gravity field, which the spherical harmonic models available at that time handled quite well. Instead it consisted of insufficient knowledge of the fine structure\* of the gravity field in the vicinity of the launch point, where the missile spends a relatively long time close to the earth and gravitationally-induced acceleration errors integrate rapidly into velocity errors that propagate for a long time into target miss. Detailed models of the gravity field in the launch region (discussed in Section 2.3.4) were accordingly introduced to cope with this problem. These models are based on closely-spaced, high-precision measurements of the gravity field magnitudes in areas surrounding the launch site. They effectively suppress ballistic missile miss contributions that might otherwise dominate the weapon system CEP.

As the foregoing discussion indicates, the geophysical information base that supports the formulation of MC&G products (see Section 2.4) constitutes an intrinsically important ingredient in modern weapon system applications. Key elements of the information base that are important to the current generation of force components include the geodetic and geophysical attributes of the launch point and target points, high-order

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\*The sensitivity of an inertial navigation system to errors in the gravitational model depends on the missile velocity and altitude. At the low velocities and altitudes characteristic of the launch region, the navigation system is particularly sensitive to short-wavelength gravitational modeling errors. At the higher velocities and altitudes characteristic of the free-flight phase, the navigation system is sensitive to gravitational errors at the long wavelengths represented in the global gravity models. This is more fully explained in subsequent chapters of this Unit.

global gravity models, and detailed models of the fine structure of the gravity field in the vicinity of the launch point. Emerging generations of weapon systems will require even more MC&G support data, as advanced navigation systems employing such techniques as correlation guidance come into widespread use. The purpose of this unit is to describe in some detail the applications of these various geophysical data elements in the operations of modern weapon systems.



## CHAPTER TWO

### WORLD GEODETIC SYSTEMS

Attempts to extend the major local geodetic datums, like the North American Datum, the European Datum, the Indian Datum, and the Tokyo Datum (review Section 2.5 of Unit One) into a unified world system have not been successful at the level of accuracy required for military purposes. For this reason, the Department of Defense (DoD), in the late 1950s, began to develop a world system to which individual geodetic datums could be referred. Efforts of the Army, Navy, and Air Force were combined to produce the DoD World Geodetic System 1960 (WGS 60). WGS 60 was computed by the use of a combination of available surface gravity data, astrogeodetic data, and results from HIRAN\* and Canadian SHORAN\* surveys. These were used to obtain a best-fitting ellipsoid for the major datum areas and an earth-centered orientation for each of the preferred systems.

The reader will recall (Section 2.5 of Unit One) that these datums are locally oriented with respect to the geoid, and are not related in an absolute sense to the center of the earth. Figure 2.2-1 reviews these relationships. The sole contribution of satellite data to the development of WGS 60 was to provide a value for the ellipsoid flattening.

Before the development of WGS 60, the Army and Air Force had each developed a world system by using different approaches to gravimetric datum orientation (Fig. 2.2-2). To determine their gravimetric orientation parameters, the Air Force used the mean of the differences between the gravimetric

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\*Refer to Chapter Two of Unit One.

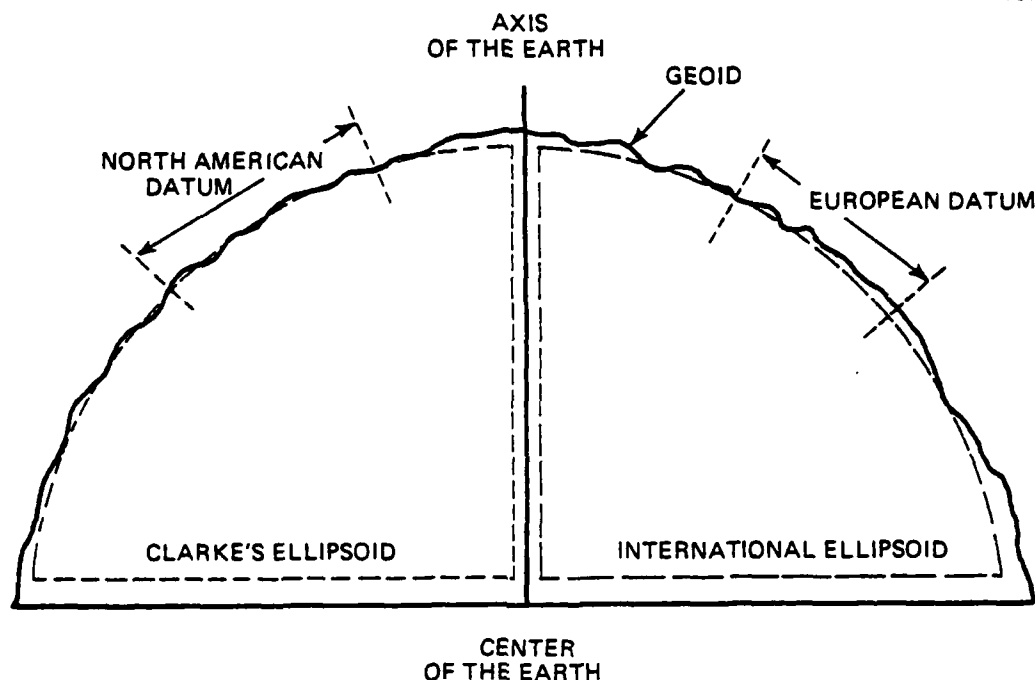


Figure 2.2-1 Local Datums

and astro-geodetic deflections and geoid heights (undulations) at specifically selected stations in the areas of the major datums. The Army performed an adjustment to minimize the difference between astrogeodetic and gravimetric geoids. The preferred datums were tied to an absolute earth-centered coordinate frame by matching the relative astro-geodetic geoids of the preferred datums with an earth-centered gravimetric geoid. Since the Army and Air Force system agreed remarkably well for the North American, European, and Tokyo Datum areas, they were consolidated to constitute the WGS 60.

#### 2.2.1 World Geodetic System 1966

In January 1966, a World Geodetic System Committee was charged with the responsibility of developing an improved WGS needed to satisfy mapping, charting, and geodetic requirements. Additional surface gravity observations, results from

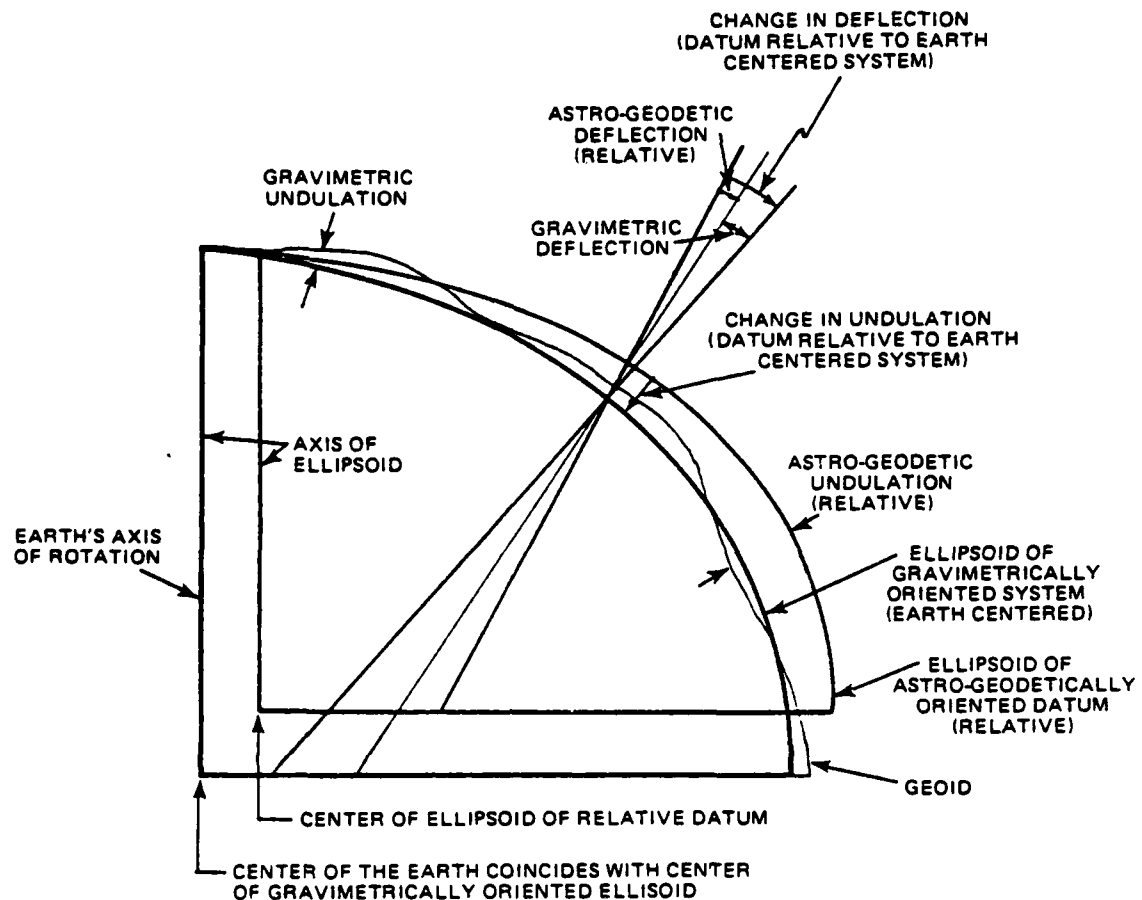


Figure 2.2-2 Methods of Datum Orientation

the extension of triangulation and trilateration networks, and a large volume of Doppler and optical satellite data had become available since the development of WGS 60. With the use of the additional data and improved computational techniques, WGS 66 was produced. It served DoD needs for about five years after its implementation in 1967. The defining parameters of the WGS 66 Ellipsoid were the flattening ( $1/298.25$ ), determined from satellite data, and the semimajor axis ( $6378145$  m), determined from a combination of Doppler satellite and astro-geodetic data involving a geoid-match technique. A worldwide  $5 \text{ deg} \times 5 \text{ deg}$  mean free air gravity anomaly field provided the

basic data for the WGS 66 gravimetric geoid. Also, a geoid referred to the WGS 66 Ellipsoid was derived from available astrogeodetic data to provide a detailed representation within certain limited land areas.

#### 2.2.2 World Geodetic System 1972

After an extensive effort, extending over a period of approximately three years, the WGS Committee completed the development of the Department of Defense (DoD) World Geodetic System 1972 (WGS 72). Selected satellite, surface gravity, and astrogeodetic data available through 1972 from both DoD and non-DoD sources were used in a Unified WGS Solution (a large scale least squares adjustment). The results of the adjustment consisted of corrections to initial station coordinates and coefficients defining the gravitational field.

The largest collection of data ever used for WGS purposes was assembled, processed, and applied in the development of WGS 72. Both optical and electronic satellite data were used. The electronic satellite data consisted, in part, of Doppler data provided by the U.S. Navy and cooperating non-DoD satellite tracking stations established in support of the Navy's Navigational Satellite System (NNSS). Doppler data were also available from numerous GEOCEIVER<sup>\*</sup> sites established during 1971 and 1972. Doppler data were the primary data source for WGS 72. Additional electronic satellite data were provided by the SECOR<sup>\*\*</sup> (Sequential Collation of Range) Equatorial Network completed by the U.S. Army in 1970. Optical satellite data

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\*A navigation receiver, to be discussed in detail in Unit Four.

\*\*An electronic ranging instrument used for geodetic purposes (refer to Section 2.2 of Unit One).

from the Worldwide Geometric Satellite Triangulation Program were provided by precision telescopic camera systems such as the BC-4 and PC-1000. Data from the Smithsonian Astrophysical Observatory included telescopic camera (Baker-Nunn) and some laser ranging data.

The surface gravity field representation used in the unified WGS solution consisted of a set of 410 (10 deg  $\times$  10 deg) equal-area mean free-air gravity anomalies determined solely from terrestrial data. This gravity field model includes mean anomaly values compiled directly from observed gravity data wherever available in sufficient quantity. The values for areas of sparse (or no) observational data were developed from geophysically compatible gravity approximations using gravity-geophysical correlation techniques. Approximately 45 percent of the 410 mean free air gravity anomaly values were determined directly from observed gravity data.

The basic form of the astrogeodetic data is the components of the deflection of the vertical, referred to the various local geodetic datums. These deflection values were integrated into astrogeodetic geoid charts referred to these local datums. The geoid heights contributed to the unified WGS 72 solution by providing additional (and more detailed) data for land areas. Conventional ground survey data were included in the solution to enforce a consistent adjustment of the coordinates of neighboring observation sites of the BC-4, SECOR, Doppler, and Baker-Nunn systems. Also, eight geodimeter<sup>\*</sup> long-line precise traverses were included, for the purpose of controlling the scale of the solution.

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\*An electro-optical distance measuring device (refer to Section 2.2 of Unit One).

The Unified WGS 72 Solution was a solution for geodetic positions and associated parameters of the gravitational field based on an optimum combination of available data. The WGS 72 ellipsoid parameters, datum shifts, and other constants were determined as part of a separate computation process. For the unified solution, equations were formed, based on the data sets mentioned in the preceding paragraph, and solved to obtain positions and gravity parameters.

The value for the semimajor axis ( $a$ ) of the WGS 72 Ellipsoid is 6378135 m. The adoption of a semimajor axis value 10 meters smaller than that for the WGS 66 Ellipsoid was based on several calculations and indicators. One of the more extensive calculations involved the combination of satellite and surface gravity data to determine positions and spherical harmonic coefficients of the gravitational field.

There are two variations of this general procedure, differing principally in that one does not involve a gravitational field determination. Using each procedure (and a considerable amount of computer programming) various sets of satellite-derived station coordinates and gravimetric deflection of the vertical and geoid height data produced estimates of local-to-geocentric datum shifts, datum rotation parameters, a datum scale parameter, and a value for the semimajor axis of the WGS 72 Ellipsoid.

Eight solutions were made with the various sets of input data, both from an investigative point of view and also because of the limited number of unknowns which, due to computer program limitations, could be included in any individual solution. Doppler satellite tracking and astro-gravimetric datum orientation stations were used on a selected basis in the various solutions. In these eight solutions, the input

values for the semimajor axis and flattening of the ellipsoid were 6378145 m and  $1/298.26$ , respectively. Different combinations of Doppler satellite tracking and astro-gravimetric datum orientation stations provided values for the semimajor axis ranging from 6378134.5 to 6378137.2 m. Also, eight additional solutions were made in which the only change was in the input value for the ellipsoidal semimajor axis. Using 6378130 m for this input parameter, the solutions for the semimajor axis ranged from 6378133.6 to 6378136.6 m. Based on these results and those from other related studies carried out by the WGS Committee, the value of the semimajor axis,  $a = 6378135$  m, was adopted. The value adopted for the flattening was  $1/298.26$ .

In the development of local-to-WGS 72 datum shifts, results of various geodetic techniques were investigated, analyzed, and compared. The redundancy of techniques and data provided assurance that the system accepted as WGS 72 was the best attainable using the methods and data available in 1972. The variations between results obtained from the different methods were helpful in assigning accuracy levels to the adopted datum shifts. Those shifts adopted were based primarily on a large number of previously determined Doppler TRANET and GEOCEIVER station coordinates, which were available worldwide.

Efforts are currently under way toward the development of a new world geodetic system to be called WGS 82. Details are beyond the scope of this text.

### CHAPTER THREE

#### INERTIAL NAVIGATION AND GUIDANCE

Inertial navigation systems (INS) are used on modern aircraft, missiles, and submarines because of their capability for providing accurate position and velocity information without the need to receive any radiated energy. This autonomous feature is particularly attractive for military vehicles because it is impossible to jam or spoof. The only information required by the system is accurate measurement of the host vehicle's acceleration. This is illustrated in Fig. 2.3-1.

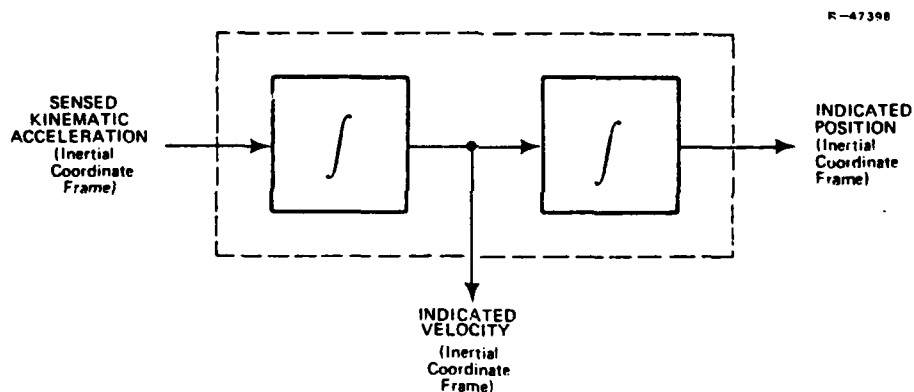


Figure 2.3-1 Idealized Inertial Navigation System

At this point it is useful to distinguish among surveys, navigation, and guidance. Surveys, discussed in Unit One, are measurement data sets taken to determine and record the quantitative description of physical features (terrain elevations, gravity field, physical boundaries, etc.). Operationally the features and the description are of primary interest. Issues such as the means of gathering the data and the locations of the instruments\* and survey vehicles are secondary.

\*For example, the location of a survey benchwork is more important than where a particular theodolite may be stationed.



Navigation, however, is primarily concerned with knowing the position and velocity of an observer (and often his vehicle). The detailed nature of the observer's surrounding is usually of lesser importance. One implication of the difference between navigation and survey manifests itself in the way data are processed. Often, in the course of the survey, only the minimal amount of processing which can be supported by a field computer is performed as the survey proceeds (just enough to provide precautions against blunders and to serve the ongoing needs of the survey party). Later (post-mission) all of the data are used to extract the greatest possible amount of information about each survey point. With navigation, however, the luxury of post-mission processing is not available; the navigation outputs are usually needed as the vehicle proceeds. Consider, for example, how useless an automobile speedometer would be which provided an exceedingly accurate record of speed as a function of distance traveled -- but only after the completion of the trip, i.e. no real time information. Navigation outputs must be obtained in the course of vehicle operation. Note that the penalty for real time availability of the data is lower accuracy than can be achieved with post-mission processing.

Guidance is the process of determining and implementing the velocity and position schedule which takes a vehicle from its current state<sup>\*</sup> to a desired state. Another automobile-related example is useful. A motorist traveling at 45 mph on a divided highway begins passing a truck at the crest of a hill. Because the truck will likely increase speed on the coming downgrade, the driver chooses to accelerate to 55 mph so that the passing maneuver will be completed quickly.

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\*Most easily visualized as a specification of position and velocity.

Expressing this operation in terms of guidance concepts, the current state of the automobile is 45 mph. Based on sensory data (truck's presence, road conditions, etc.), instrument outputs (45 mph speedometer reading), and prestored information (driver's experience), the guidance computer (driver's brain) decides that a new velocity state (55 mph) is desired. The generated guidance command (velocity to be gained is 10 mph) results in a control output being implemented (driver's foot depresses accelerator pedal further). In this example, involving only the vehicle's speed (a scalar), the guidance is particularly simple; in systems such as ICBMs, guidance computers must continually calculate the required vector velocity to be gained so that, at the end of powered flight, the payload will have the correct velocity vector from its then-current position to impact the desired target. Note that conceptually the navigation solution is prerequisite to solving for guidance commands.\* Put another way, "navigation tells you where you are, guidance tells you how to get where you want to go."

The intent of the following discussion is to highlight some of the more important aspects of inertial navigation, leaving the subtleties for later study. A reading list of works containing more advanced treatments is provided at the end of the Unit.

### 2.3.1 Mechanization

Inertial navigation systems consist of instruments designed to measure a vehicle's acceleration continuously and to integrate these measurements. The result of one integration is velocity; the second integration yields position. The

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\*Often the same INS solves both for navigation and guidance variables.

constants of integration are the vehicle's velocity and position when the navigation starts.

Because acceleration measurements are made with respect to inertial space,\* and since most navigation problems of interest involve distances and velocities referred to the earth's surface (or to bodies of the solar system), the relative motions between inertial coordinates and the coordinate system in which the acceleration measurements are made must be taken into account. These coordinate frames are usually accelerating with respect to one another. Hence a variety of transformations and corrections must be applied to INS-sensed accelerations before processing. In particular, inertial systems are mechanized to account for the Coriolis acceleration resulting from rotation of the coordinate system of the accelerometer measurements with respect to the inertial coordinate frame.

A second correction must be applied to cancel the portion of sensed acceleration due to gravity. The necessity for such a correction is easily understood by considering the acceleration sensed by an INS "anchored" (by some unspecified means) with respect to inertial space. If the system is near enough to a planet for its gravity field to be sensed by the accelerometers, then the accelerations due to gravity would, if not accounted for in the INS mechanization, integrate to erroneous velocity and position changes. This would occur despite the fact that the true velocity is zero and the position is unchanged. Gravity compensation is very important in high-accuracy inertial systems. This fact has been alluded to earlier and is discussed later in more detail.

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\*Most easily conceptualized (neglecting relativistic considerations) as a coordinate frame fixed with respect to the "fixed" stars (e.g., Section 2.4 of Unit One).

In addition to sensing acceleration<sup>\*</sup> it is necessary for an INS to measure the angular orientation of its accelerometers with respect to inertial space.<sup>\*\*</sup> Otherwise it is impossible to distinguish between translational and rotational motion. An example is instructive.

Suppose that three rigidly-connected, orthogonally-aligned accelerometers are situated in inertial space as illustrated in Fig. 2.3-2. If accelerometers  $a_x$  and  $a_y$  each register the same positive output, that output could be caused either by translational acceleration along line OP or by rotation at constant angular velocity about the z axis. (Ambiguous combinations of rotation and translation are also possible.) Thus a separate means for measuring rotation of the accelerometer assembly is needed.

Gyroscopes, because of their tendency to remain oriented toward a particular direction in inertial space, are used to provide the coordinate frame angle information needed to resolve the accelerometer measurements. Although a description of different gyro types and operating details is beyond the scope of this course, it is appropriate to mention several gyro mechanizations commonly used in inertial navigation and guidance systems. Each mechanization usually involves three gyros, each measuring one of the angular degrees of freedom of the system.<sup>†</sup>

\*INSs usually have three accelerometers to provide the three components of a vehicle's acceleration vector. Exceptions occur for vehicles that are constrained to operate very close to the earth's surface. In such applications the vertical channel of the INS is sometimes omitted.

\*\*Or other coordinate frame of interest.

†Some INSs employ two so-called "two-degree-of-freedom gyros." The redundant gyro output is usually employed as an error check or monitor mechanism.

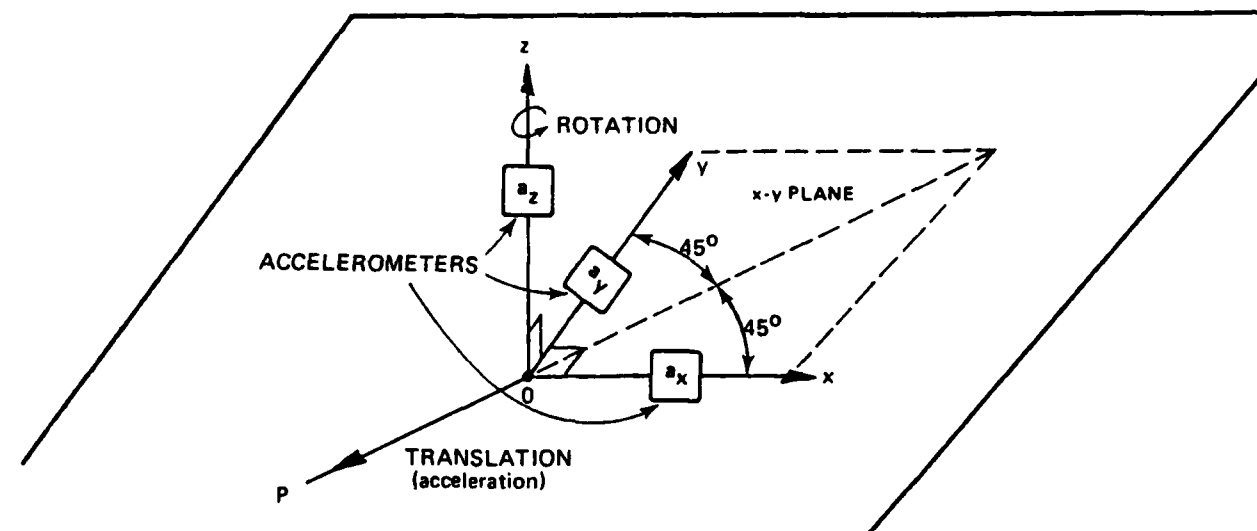


Figure 2.3-2 Accelerometer Response to Rotation and Translational Acceleration

Strapdown - As the name implies, the accelerometers and gyros in a strapdown mechanization are rigidly attached to the vehicle's frame. When the vehicle maneuvers, the gyro outputs provide instantaneous values of the angles and angular rates between the coordinate frame of the accelerometer axes and inertial space.\* These angles and rates are used to resolve the accelerometer outputs and integrate them into inertially-measured velocity and position. Because the strapdown mechanization requires accelerometers which must operate without any isolation from vehicle motion, INS accuracy is usually not as good as with inertial systems that take advantage of the gyros

\*Or other coordinate frame in which it might be desirable to perform the integrations.

to provide some measure of angular isolation from vehicle movement. Such isolation mechanizations are described below. Historically, another limitation of strapdown systems has been the burden of performing, in real time, the coordinate resolutions and integrations of sensed acceleration. More recently the advent of microprocessors and large scale integrated digital circuitry has made computer limitations less important in strapdown inertial systems. Particular advantages of strapdown systems are small size and low cost. These benefits occur because no mechanically-stabilized platform element is involved -- in contrast to the other mechanizations described below. Strapdown inertial systems find frequent application in tactical missiles.

Space-Stable - Certain types of high-accuracy gyros, such as those with electrostatically suspended rotors, are most advantageously operated on a platform which is maintained at a fixed orientation with respect to inertial space. This is accomplished by maintaining the gyro cases (attached to the platform) at a constant attitude with respect to the spin axis of the rotor. (Recall that the rotor axis maintains itself at a fixed angular orientation in inertial space.) The fixed inertial orientation of the platform, and hence the accelerometers, is maintained (regardless of changes in vehicle attitude) by a series of connected bearing assemblies called gimbals. Servo motors, driven by gyro outputs generated whenever the vehicle rotates with respect to inertial space, drive the gimbals and provide the mechanism for maintaining the inertially-fixed platform orientation. Since the accelerometers, because of their location on the stabilized platform, are not subject to vehicle rotations, better accelerometer performance (i.e. less error) can usually be achieved with space-stable systems than with strapdown mechanized inertial systems. Because space-stable systems are usually of very high accuracy, they are

typically found in strategic weapon systems such as inter-continental missiles and long range bombers.

Local-Level - Local-level systems also involve the "stable platform" concept but, as the name suggests, the platform is maintained in alignment with coordinates defined by the vehicle's position. Frequently the coordinates are such that two of the accelerometers are aligned horizontally and one is vertical. To maintain the platform in a level attitude as the earth turns and/or as the vehicle moves, the initial space pointing axis of the gyros is continuously changed by precisely computed amounts. These computations are based on stored values of the earth's angular rate and the INS-computed values of velocity with respect to the earth's surface (i.e. groundspeed). The gyro pointing changes are accomplished by applying calibrated torques to the gyro rotors for controlled lengths of time. In response to the torque, the gyro precesses through the desired angular change in pointing direction. Gyros intended for local-level mechanization applications must have this "torquing" capability.

A one-dimensional illustration of a local-level mechanization is presented in Fig. 2.3-3. Earth rotation is omitted from the figure. The horizontal accelerometer senses the vehicle's acceleration along the great circle shown. As a differential distance  $\Delta C$  is traversed, a platform gimbal angle adjustment,  $\Delta\theta$ , is made by torquing the gyros where

$$\Delta\theta = \frac{\Delta C}{R} \quad (2.3-1)$$

Thus, the level platform attitude is maintained as the vehicle moves.

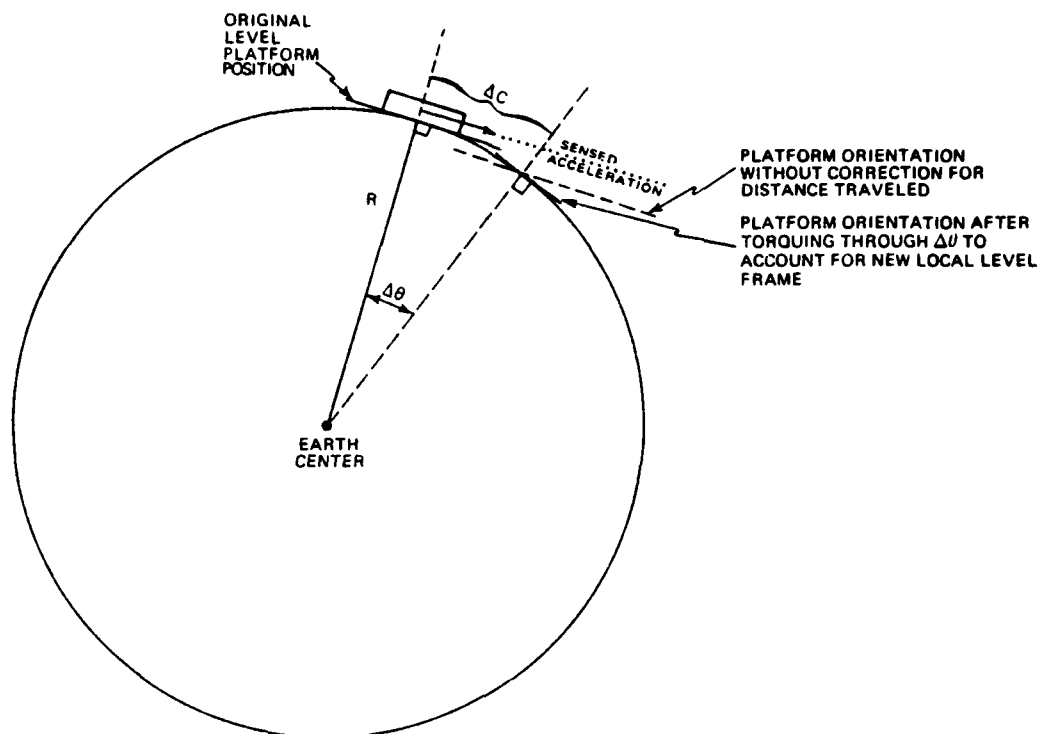


Figure 2.3-3 Local-Level Mechanization in One Dimension

Key advantages of the local-level mechanization include constant accelerometer orientation in the gravity field<sup>\*</sup>, benign dynamical behavior of certain inertial system error sources (such as accelerometer bias), and easier requirements for interfacing with other systems. Often, systems used in conjunction with an INS involve position or velocity information expressed with respect to terrestrial (geographic) coordinates. This is particularly true for marine navigation where local-level mechanized systems enjoy widespread use.

<sup>\*</sup>Accelerometer calibration parameters are less likely to change if the instrument is maintained at a constant attitude with respect to the gravity vector, resulting in higher accuracy.



### 2.3.2 Error Theory<sup>(†)</sup>

Because inertial systems integrate acceleration signals to velocity and then to position outputs, gyro drift bias and noise in the sensed acceleration cause growing errors in the indicated velocity and position. For this reason inertial system accuracy is usually given in terms of a benchmark time. This is usually the time span for a given level of error to develop following a system update from a specified external data reference. The need to bound INS error growth also motivates the use of external navigation aids. Some navigation aids are discussed in Section 2.3.7. Because of the inherent instability of INS errors and because of the exacting military applications for which inertial system are used, considerable effort has been devoted to understanding how inertial system errors propagate, learning their dynamics, and using the broadest range of possible techniques to mitigate them.

Among the interesting and useful conclusions from the study of inertial system theory is that INS errors propagate with the same dynamics regardless of the manner in which the system is mechanized (space-stable, strapdown etc.).<sup>\*</sup> As a result, the propagation of errors through any inertial system can be described by the same set of equations.<sup>\*\*</sup> Only the characterization of the error sources requires attention to mechanization details. Although surprising at first, the applicability

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(†)This section contains material at a more advanced level than the rest of the text.

\*Of course it is presumed that the mechanization is "dynamically exact." That is, if the INS sensors are perfect (i.e. without error) and no environmental errors are present, then the INS-indicated velocities and positions would be correct.

\*\*However, the error sources, themselves, do depend upon the frame of the sensor. See next section.

of the same set of error equations to all inertial systems should be easy to understand from the point of view of Fig. 2.3-1. All inertial navigation systems, regardless of mechanization, are intended to integrate sensed acceleration into indicated velocity and position. Since the dynamics of this integration is the same for any type of system (they all must implement Newton's laws of motion), it is to be expected that errors in quantities that drive the system should propagate in similar, mechanization-invariant fashion.

Another way of stating this observation is to note that both the INS mechanization and error equations can be written in completely vectorized form. Vector equations can be expressed in any coordinate system (they are reference-frame invariant); this mathematical description provides the basis for arbitrary mechanization reference frames as well as for INS error propagation descriptions which are reference-frame independent. Examples of vector formulations of INS mechanization and error equations are provided in Section 2.3.6.

This property of coordinate frame invariance of the INS mechanization and error equations is employed to frequent advantage particularly in the analysis of the effects of geophysical and geodetic (G&G) errors on inertial systems. Specific examples are provided in Section 2.3.6. Since G&G errors are usually most conveniently expressed in a north-pointing, local-level frame, this frame is often used.

Sensor Reference Frame - It is appropriate to emphasize that, while the INS error equations may be cast in a form which is reference-frame independent (vector equations), this is not the case for the sensor models. The sensor measurements actually occur in the reference frame of the sensor. If the errors

in these measurements are to be modeled in some other, conveniently chosen frame, a transformation is required to relate the sensor reference frame to the chosen frame. This is illustrated by considering bias vertical accelerometer error in a local-level INS for the case of a stationary vehicle located on the earth's equator (depicted in Fig. 2.3-4a). In the local level frame this error may be modeled correctly as a constant. Consider the same situation but now with the accelerometer located on a space-stabilized inertial platform (axis fixed with respect to the stars) as shown in Fig. 2.3-4b. The accelerometer error expressed in the local level frame is now sinusoidally varying at the earth rotation frequency since, to an observer in the local level frame, the accelerometer will be tumbling at the earth rotation rate.

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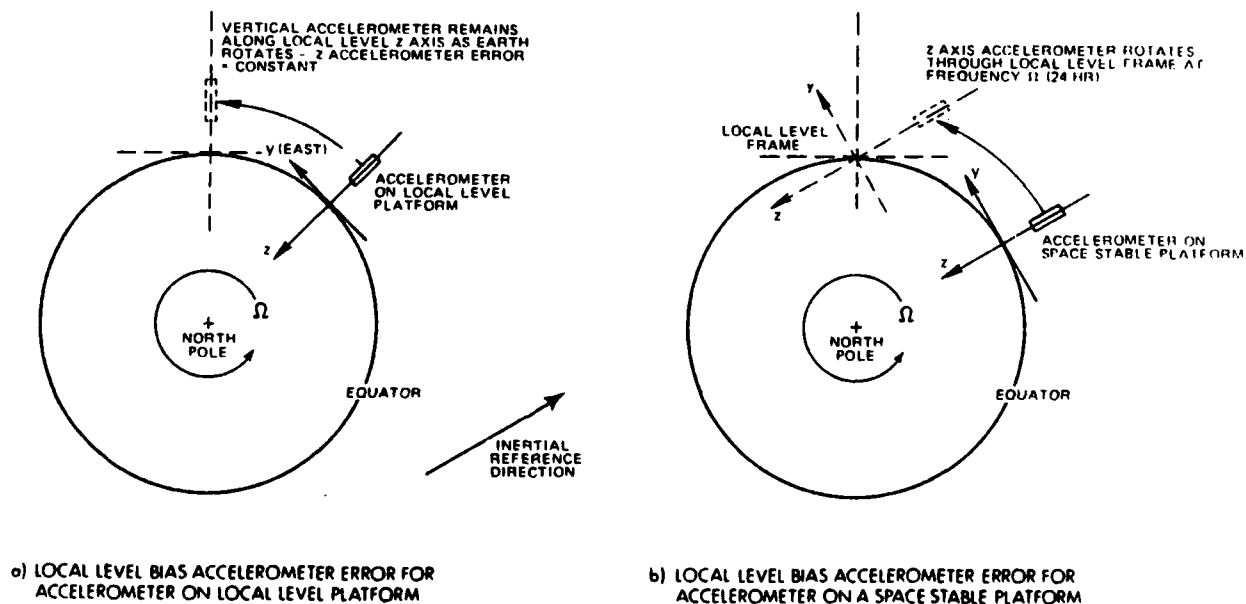


Figure 2.3-4

Error from Accelerometer Bias Expressed in Local-Level Coordinates from Both Space Stable and Local-Level Accelerometer Mechanizations

Tilt Errors - Errors in an inertial platform's orientation result in misresolution of sensed accelerations and hence velocity and position errors. Two characterizations of inertial platform orientation errors are in common use. One considers the angular difference between an inertial platform's actual orientation and the orientation it would have if there were no errors. This quantity is called platform tilt error and it is frequently denoted by the vector angle,  $\underline{\phi}$ . Note that  $\underline{\phi}$  has three components corresponding to the three angular degrees of freedom of a rigid body. Platform tilt angles are particularly useful when system calibrations or updates are being made which require a physical adjustment to the inertial platform (or to a computer variable accounting for platform misalignment). In modern, high quality inertial systems the  $\underline{\phi}$  angles are very small, usually of the order of several  $\widehat{\text{sec}}$  or better.

A second quantity used to describe inertial system platform errors is the angular difference between the inertial platform's actual orientation and the orientation it would have in the absence of errors at the currently computed position. This quantity is called computer platform error and represents the tilt of a fictitious platform which is defined by the actual platform tilt,  $\underline{\phi}$ , and the position error. The definition is illustrated for a local-level system in Fig. 2.3-5. The computer platform error is frequently represented by the vector angle,  $\underline{\psi}$ .<sup>\*</sup> In a high-quality INS, the magnitude of  $\underline{\psi}$  is also of the order of several  $\widehat{\text{sec}}$  or less. A third quantity relates the two, namely the portion of the computer platform error resulting from position error,  $\underline{\theta}$ . Figure 2.3-5 illustrates the relation between  $\underline{\theta}$ ,  $\underline{\phi}$ , and  $\underline{\psi}$ ; namely

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<sup>\*</sup>Not to be confused with reduced latitude, used in geodesy, which is also represented by the symbol  $\psi$ .

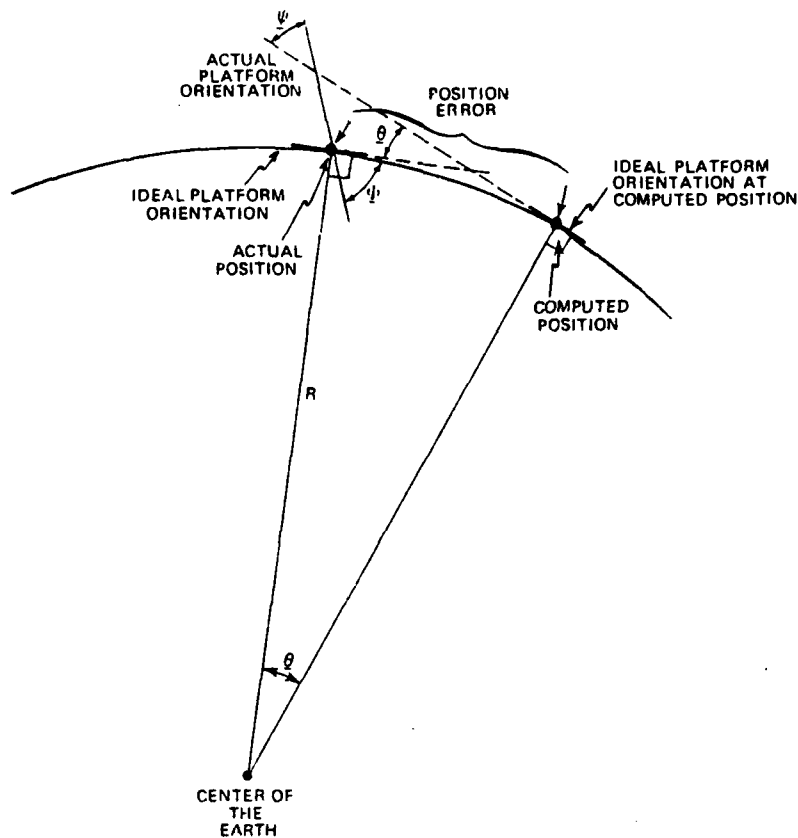


Figure 2.3-5 INS Tilt Angle Diagram

$$\psi = \phi - \theta \quad (2.3-2)$$

At first thought it seems artificial to define a tilt error ( $\psi$ ) in terms of two other errors. However, the advantage of the  $\psi$  representation is that it allows the equations which describe INS error propagation to take on a particularly simple form in which the tilt error equations decouple from the position and velocity error equations. The decoupled equations can be solved for the  $\psi$  angles, which in turn are driving terms for the position and velocity error equations. This is illustrated in Fig. 2.3-6. Some insight is gained into the tilt error propagation dynamics for the case of a vehicle traveling near the earth's surface at low latitude with a speed significantly less than 450 m/sec (approximate equatorial velocity of

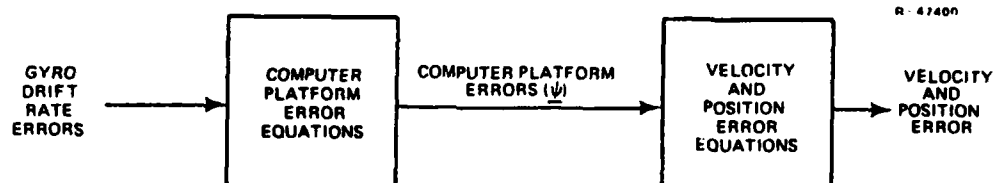


Figure 2.3-6 Inertial System Error Diagram

the earth's surface with respect to inertial space). If the gyro drift rates and  $\psi$  angles are expressed in polar (P), equatorial (E), and longitude ( $\lambda$ ) coordinates as illustrated in Fig. 2.3-7, the computer platform tilts are given by

$$\frac{d\psi_P}{dt} = \varepsilon_P \quad (2.3-3)$$

$$\frac{d\psi_E}{dt} = \Omega \psi_\lambda + \varepsilon_E \quad (2.3-4)$$

$$\frac{d\psi_\lambda}{dt} = -\Omega \psi_E + \varepsilon_\lambda \quad (2.3-5)$$

where:

$t$  = time

$\varepsilon_E, \varepsilon_\lambda, \varepsilon_P$  = components of gyro drift

$\psi_E, \psi_\lambda, \psi_P$  = computer platform tilt angles  
(components of  $\psi$ )

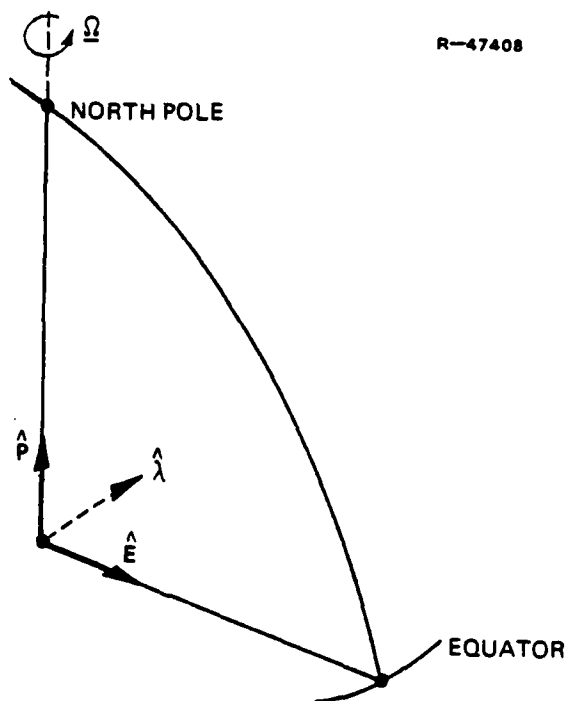


Figure 2.3-7 Polar, Equatorial, and Longitude Coordinates

In Fig. 2.3-7 the unit  $\hat{\lambda}$  vector is defined in terms of the  $\hat{P}$  and  $\hat{E}$  unit vectors by

$$\hat{\lambda} = \hat{P} \times \hat{E} \quad (2.3-6)$$

and the earth's angular velocity vector,  $\underline{\Omega}$ , is given by

$$\underline{\Omega} = 0 \hat{E} + 0 \hat{\lambda} + \Omega \hat{P} \quad (2.3-7)$$

Equations 2.3-3 through 2.3-5 are shown in block diagram form in Fig. 2.3-8. Note that the equatorial and longitude gyro drift components result in undamped oscillations with a 24-hour period and that the polar gyro drift component causes linear growth to occur in the polar component of computer platform error.

Schuler Loop Errors - In a local-level solution frame,\* the north (N), east (E), and vertical (Z-down) components of position error for an inertial system operating near the earth's surface are given by

$$\frac{d}{dt} (\delta P_N) = \delta V_N - \frac{V_E}{R} \tan \phi \delta P_E - \frac{V_N}{R} \delta h \quad (2.3-8)$$

$$\frac{d}{dt} (\delta P_E) = \delta V_E + \frac{V_E}{R} \tan \phi \delta P_N - \frac{V_E}{R} \delta h \quad (2.3-9)$$

$$\frac{d}{dt} (\delta h) = -\delta V_Z + \frac{V_N}{R} \delta P_N + \frac{V_E}{R} \delta P_E \quad (2.3-10)$$

where:

$\delta h$  = altitude error

$\delta V_N, \delta V_E, \delta V_Z$  = velocity error components

$V_E, V_N$  = the east and north velocity components

$R$  = the earth's radius

$\phi$  = latitude

These equations come from examining the mathematical effect of small variations from nominal (perturbation analysis) of the quantities in the inertial system mechanization equations. The velocity error equations are

$$\begin{aligned} \frac{d}{dt} (\delta V_N) = & \mu_N - (A_Z + g) \psi_E + A_E \psi_Z - \frac{g}{R} \delta P_N + \delta g_N \\ & - (2 \Omega \sin L + \frac{V_E}{R} \tan \phi) \delta V_E + \frac{V_N}{R} \delta V_Z \end{aligned} \quad (2.3-11)$$

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\*Note the distinction between the (N,E,Z) reference frame and the (E, $\lambda$ ,P) frame discussed above.



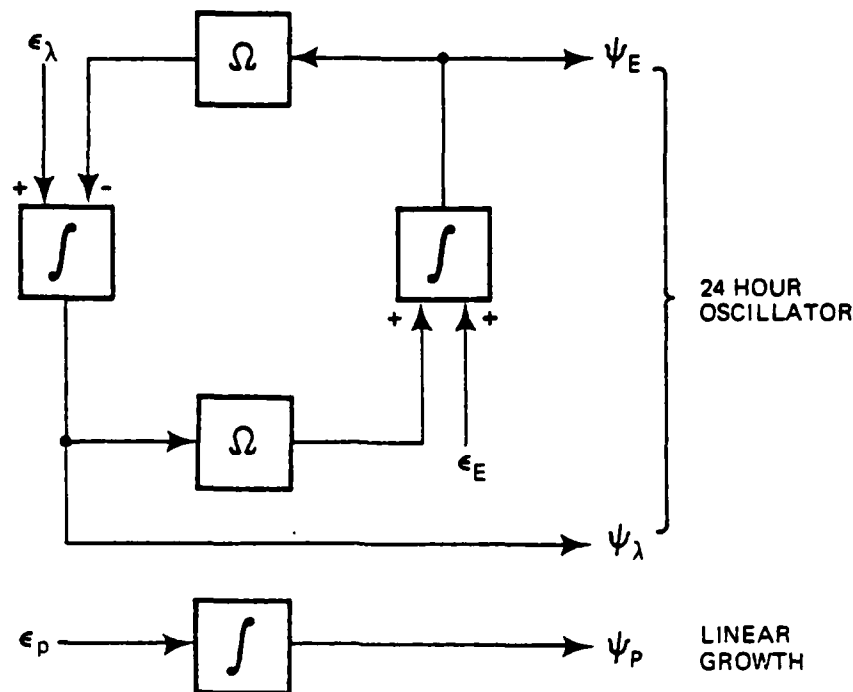


Figure 2.3-8 INS Computer Platform Tilt Error Dynamics

$$\begin{aligned}
 \frac{d}{dt} (\delta V_E) = & \mu_E - (A_Z + g) \psi_N + A_N \psi_Z - \frac{g}{R} \delta P_E + \delta g_E \\
 & + (2 \Omega \sin \phi + \frac{V_E}{R} \tan \phi) \delta V_N + (2 \Omega \cos \phi + \frac{V_E}{R}) \delta V_Z
 \end{aligned}
 \tag{2.3-12}$$

$$\begin{aligned}
 \frac{d}{dt} (\delta V_Z) = & \mu_Z + A_N \psi_E + A_E \psi_N - \frac{2g}{R} \delta h + \delta g_Z - \frac{V_N}{R} \delta V_N \\
 & - (2 \Omega \cos \phi + \frac{V_E}{R}) \delta V_E
 \end{aligned}
 \tag{2.3-13}$$

where:

$g$  = the acceleration due to gravity

$\mu_N, \mu_E, \mu_Z$  = accelerometer errors

$\delta g_N, \delta g_E, \delta g_Z$  = errors in compensation for the acceleration of the gravitational field

$A_N, A_E, A_Z$  = the components of kinematic acceleration acting on the system

The dynamics of Eqs. (2.3-8) through (2.3-13) are visualized more readily by the block diagram illustrated in Fig. 2.3-9. In Fig. 2.3-9 the cross coupling terms are represented by dotted lines. Usually the dynamical effects of these terms are not as strong as the principal error response illustrated by the solid lines. Observe that the horizontal channel errors ( $\delta P_N$  and  $\delta P_E$ ) exhibit sinusoidal oscillatory response with a frequency given by

$$\omega_s = \sqrt{g/R} \quad (2.3-14)$$

This frequency is called the Schuler frequency. The corresponding period is 84.4 minutes. Note that the Schuler period is the same as that of a hypothetical earth satellite at zero altitude. The closed-loop description of INS horizontal channel position and velocity errors illustrated in Fig. 2.3-9 is referred to as a Schuler loop. It and the 24-hour oscillation rate illustrated in Fig. 2.3-8 are the most important dynamical characteristics of inertial system error behavior.

In Fig. 2.3-9 the altitude channel error loop is seen to be similar to the horizontal channel Schuler loops but with an important exception. The sign of the error signal fed back into the acceleration error node is positive. Thus the altitude channel is unstable with exponential error growth. This growth is coupled to the horizontal channels through the vertical velocity error cross coupling terms.

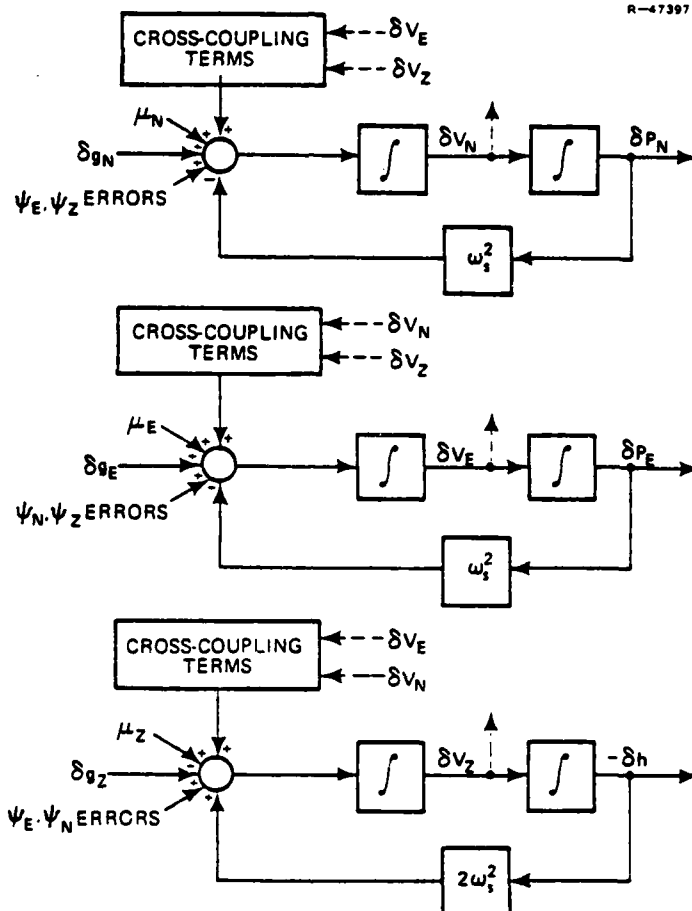


Figure 2.3-9 INS Schuler and Altitude Loop Error Dynamics

In addition to vertical channel instability, the Schuler loops will exhibit a linearly growing sinusoidal envelope if any of the error sources driving them has power at the Schuler frequency. One important Schuler frequency error source is the inability to compensate perfectly for gravity. Gravity errors are discussed at further length in the next section.

### 2.3.3 Inertial Systems and Gravity Effects

In Section 2.3.1 the need to account for the effects of gravity accelerations on an INS is described. The errors resulting from incomplete compensation of gravity vector components influence terms in the inertial system velocity error derivative Eqs. (2.3-11) through (2.3-13). This section discusses, in more detail, gravity compensation for a vehicle traveling in the vicinity of the earth.

The gravity field of the earth is conventionally approximated as that of a homogeneous ellipsoid. Recall from Unit One that the ellipsoidal approximation allows the value and direction of gravity to be computed at any point on the earth's surface, or in space, from an analytical formula. For present purposes the ellipsoidal gravity formula can be represented as

$$\underline{g}^{\text{ref}*} = \underline{g}^{\text{ref}}(\underline{r}) \quad (2.3-15)$$

where  $\underline{g}(\underline{r})$  is the gravity vector at a point in space defined by the position vector  $\underline{r}$ .

It turns out that the accuracy of the ellipsoidal gravity formula is remarkably good. The difference between the magnitude of actual gravity and that of  $\underline{g}^{\text{ref}}$  (called the gravity disturbance) is usually of the order of 50 mgal. It never exceeds several hundred mgal on the earth's surface and, at higher altitudes, is diminished even further. Noting that the mean value of  $g$  is 980 gal, the reference ellipsoid gravity formula is seen to be accurate to about one part in 20,000.

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\*The symbol  $\gamma$ , conventional in geodesy, was used to designate the normal (or reference) gravity in Unit One.

Because the reference value of gravity must be computed at the current position of an INS, the compensation of Eq. (2.3-15) is mechanized as illustrated in Fig. 2.3-10. Note that the gravity compensation signal is introduced at the accelerometer outputs. This is why gravity compensation errors appear as source terms in the equations for the derivatives of velocity error rather than, say, the computer platform error ( $\psi$ ) equations.

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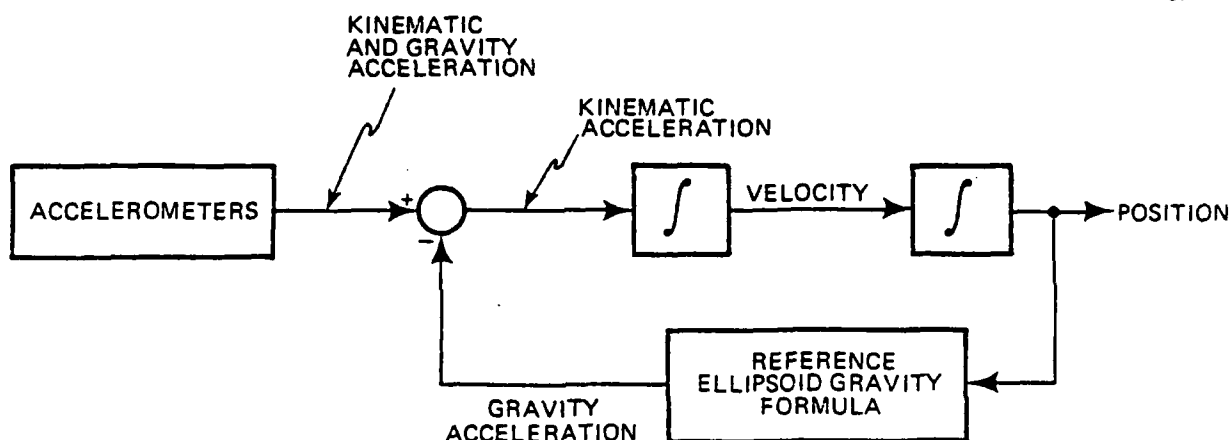


Figure 2.3-10 Reference Ellipsoid Gravity Mechanization in an INS

It is appropriate to emphasize that the gravity errors remaining after the reference ellipsoid formula compensation has been applied are simply the components of the gravity disturbance vector introduced in Unit One:

$$\delta \underline{g} = \underline{g}^{\text{true}}(\underline{r}) - \underline{g}^{\text{ref}}(\underline{r}) \quad (2.3-16)$$

where  $\delta \underline{g}$  is the gravity disturbance vector and  $\underline{g}^{\text{true}}(\underline{r})$  is the actual gravity vector at position  $\underline{r}$ . When  $\delta \underline{g}$  is expressed in local level coordinates, i.e.,

$$\delta \underline{g} = \begin{bmatrix} \delta g_N \\ \delta g_E \\ \delta g_Z \end{bmatrix} \quad (2.3-17)$$

the individual components are recognized as the gravity errors which drive the INS error Eqs. (2.3-11) through (2.3-13).

It is also appropriate to recall from Unit One the relation between the horizontal components of the gravity disturbance and the components of the deflection of the vertical, namely

$$\delta g_N = - g \xi \quad (2.3-18a)$$

$$\delta g_E = - g \eta \quad (2.3-18b)$$

where  $g$  is the value of gravity and  $\xi$  and  $\eta$  are the North and East deflection angles.

In many instances it is adequate to identify the vertical component of the gravity disturbance with the gravity anomaly,  $\Delta g$ . However, strictly speaking, this is incorrect and for exacting applications the governing relation is

$$\delta g = \Delta g + \frac{2g}{R} N \quad (2.3-19)$$

where  $N$  is the undulation of the geoid, discussed in Unit One. Equation (2.3-19) is an important relation in physical geodesy and can be derived by considering the geometry of the reference ellipsoid and true gravity fields. The detailed derivation is beyond the scope of this course.

#### 2.3.4 Other Gravity Models

Earth Gravity Model (EGM) - For very precise inertial navigation and guidance the reference ellipsoid gravity formula does not afford sufficient accuracy. One alternative approach is to express the gravitational potential field as a spherical harmonic expansion as described in Section 1.3.2 (Unit One).

$$V(r, \phi', \lambda) = \frac{GM}{r} \sum_{n=0}^{\infty} \left(\frac{R}{r}\right)^n \sum_{m=0}^n (C_{nm} \cos m\lambda + S_{nm} \sin m\lambda) P_{nm}(\sin \phi') \quad (2.3-20)$$

Notation is defined and explained in Section 1.3.2.

Since the components of the gravity vector are given by derivatives of the geopotential, the harmonic expansion formula of Eq. (2.3-20), like the reference ellipsoid formula, provides an analytical, but more accurate, means of computing gravity as a function of position. As pointed out in Chapter Four of Unit One, the coefficients  $C_{nm}$  and  $S_{nm}$  have been computed through degree and order 36 or more. It is recalled that these results are based on satellite tracking observations, surface gravimetry, and satellite altimetry data.

For inertial system gravity modeling applications, Eq. (2.3-20) is truncated to some moderate value of  $n$ , beyond which lack of confidence in the accuracy of the coefficients precludes significant improvement by adding more terms.

Note that when an EGM of the form of Eq. (2.3-20) is used, the remaining INS gravity error sources are no longer the ellipsoidally-defined gravity disturbance components of Eq. (2.3-17). Instead, the inertial system gravity error vector is given by modification of Eq. (2.3-16) to

$$\delta g^n = g^n(\underline{r})^{\text{true}} - g^n(\underline{r}) \quad (2.3-21)$$

where  $g^n(\underline{r})$  is the gravity vector computed from Eq. (2.3-20) up to degree and order  $n$ . Thus, for accurate values of the coefficients  $C_{nm}$  and  $S_{nm}$ ,  $\delta g^n$  is chiefly composed of the spherical harmonic terms from  $n+1$  to infinity.\*

Until quite recently, an important error source in EGMs was the uncertainty in the product GM. However, using new techniques, this constant has been measured to an accuracy of the order of one part in  $10^7$ . Precise data have been acquired by tracking spacecraft at very large distances from the earth and by lunar laser ranging (Section 1.4). The uncertainty in the zero degree term of Eq. (2.3-20) (i.e. the inverse square portion of the earth's gravity field) corresponding to an accuracy of one part in  $10^7$  in GM is about 0.1 mgal at the earth's surface. Because the inverse square portion of the gravity field is large and diminishes slowly with altitude, GM is an important contributor to ICBM impact errors in systems which are not mechanized using the most recent values.

Point-Mass Gravity Models - Attempts to improve upon the accuracy of EGMs require more accurate gravity field surveys in the areas where an inertial system operates. The pre-mapped gravity values are used, supplementing the EGM, as a means for further reducing the gravity errors. In the implementation of a gravity map for INS compensation, two important modeling issues must be addressed

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\*Statistical studies of gravity data have shown that the coefficients would have to be known to degree and order 3000 to compute earth gravity everywhere to an accuracy better than 1 mgal.



- How can a very large number of measurements and computations be treated efficiently?
- How is gravity computed at an unsurveyed site (e.g., part way along a missile trajectory)?

One technique that has been employed to handle these problems is to fit a fictitious set of subsurface point masses to surface gravity data (i.e., to measurements of  $\delta g^n$ ) as illustrated in Fig. 2.3-11. The gravity field due to the point masses is easily computed from the sums of the inverse square distances from each point mass. Thus gravity values at positions above the surface can be found analytically. Despite the simplicity of this analytic approach, current buried-point-mass gravity models used for INS gravity compensation involve thousands of parameters (masses) and require considerable computer time to implement. Improvements are an active area of geodesy research.

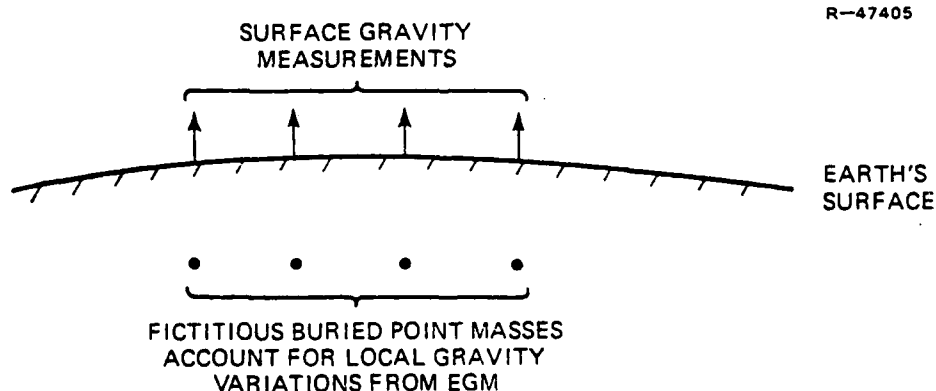


Figure 2.3-11 "Buried Point Mass" Gravity Model

One application of buried-point-mass gravity models is the generation of gravity field compensation for ICBMs. Since the high degree and order terms of Eq. (2.3-20) attenuate

rapidly with altitude (i.e., as the factor  $(R/r)^n$  becomes small) the EGM compensates trajectory points near the launch site more poorly than points further up on the trajectory. An approach which has been used successfully to model the gravity field near ICBM launch regions is to survey the surface gravity field with a survey point density that decreases with distance from the launch site. This densification variation is also extended to the positioning of the point masses. As a result, the accuracy of gravity computed from the point mass model is greatest near the launch site but degrades at distances far away. One object of the design of such models is to adjust the survey densification and point mass placement for minimum missile target impact error. Several effects that must be taken into account in the course of the optimization are gravity model errors, the attenuation of gravity compensation errors with altitude along the trajectory, and the time left for gravity errors at a given point in the trajectory to propagate into target miss. This last issue is the reason why little attention is given to the gravity field in the target area; if the missile payload encounters significant gravity errors near the target end of the trajectory, there is not sufficient time left for the gravity errors to induce appreciable position errors. One particular buried-point-mass gravity model implementation, used for the Minuteman ICBM system, is illustrated in Fig. 2.3-12. It is often referred to as the Launch Region Gravity Model (LRGM).

#### 2.3.5 Inertial System Initialization<sup>(†)</sup>

Introduction - Initialization is the process used to prepare an inertial system for mission use. Inertial systems

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(†) This section contains material at a more advanced level than the rest of the text.

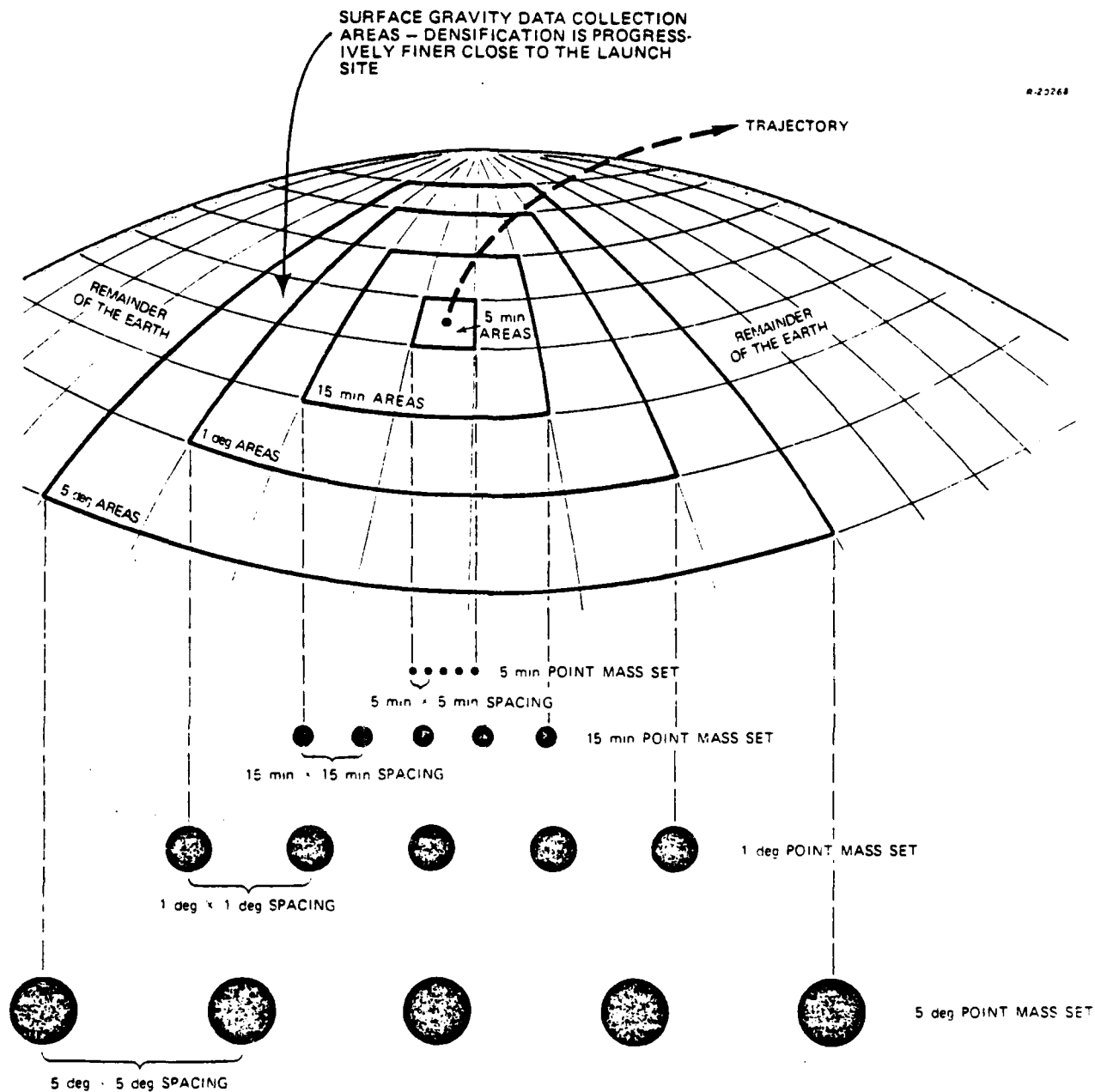


Figure 2.3-12 Launch Region Gravity Model (LRGM)

may be initialized in many different ways, depending on the demands of the specific mission. However, all initialization processes can be represented by a single, generalized mechanization block diagram, such as that shown in Fig. 2.3-13, and can be discussed in terms of three distinct activities:

- Insertion of initial values for velocity and position
- Alignment of the inertial sensors with respect to the chosen navigation coordinate system
- Calibration of the system, to determine scale factor and bias corrections for the inertial sensors, prior to using it for navigation purposes.

As indicated by Fig. 2.3-13, the initialization of an inertial system requires the availability of information derived from sources external to the inertial system. This information must bear some definable relationship to the natural outputs of the inertial system (acceleration, velocity, position, and attitude). It may be derived either from constraints on the motion of the vehicle during the initialization process or by the use of measuring equipment external to the inertial navigation system (INS).

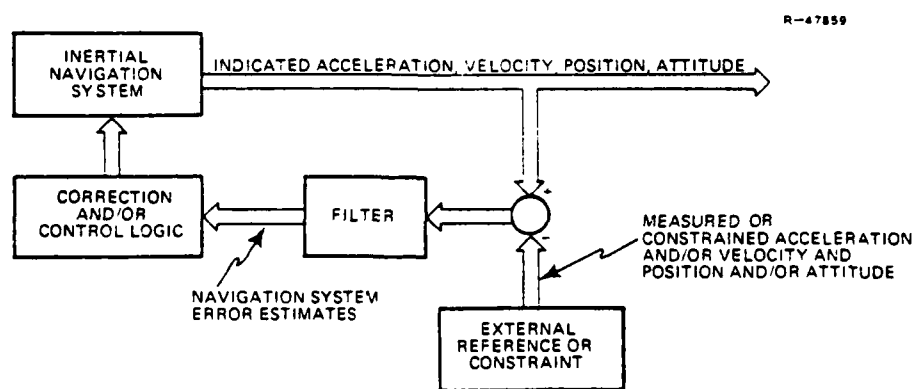


Figure 2.3-13 Generalized Inertial System Initialization Mechanization

The initialization process consists of comparing the indicated outputs of the inertial system with corresponding externally-derived information. The difference values thus obtained, via a filter and correction/control logic, are used to bring the inertial system into close correspondence with the true dynamic situation of the vehicle.

Of the three activities characterizing the initialization process, the first two (insertion of initial values for velocity and position, and alignment to an appropriate coordinate frame) are necessary in all situations. Calibration of the inertial system, on the other hand, is necessary only in those situations in which the long-term stability characteristics of the inertial sensors (i.e., the gyros and accelerometers) are not adequate for mission performance.

Insertion of Initial Values - Inertial navigation depends upon the integration (over time) of acceleration to obtain velocity and position of the vehicle. As in any integration process, the initial values of the integrals (that is, velocity and position) must be known. These initial conditions must be entered into the inertial system's computer along with appropriate trajectory guidance information.

Alignment - In general, alignment of the inertial system is a process involving the measurement of systematic error propagation patterns resulting from misalignment in the presence of noise from the inertial system itself and from the sources of external reference information. As such, it requires a finite filtering time for its successful completion and the availability of the external reference information during the alignment period.

Calibration - In some situations it is possible to conduct limited, but useful, system calibration concurrent with the alignment process. In general, however, complete inertial system calibration for a chosen mission is a process involving filtering times longer than those required for alignment and characterized by special calibration programs and procedures.

Two further points are noted prior to discussion of the various forms taken by the initialization process:

- Inertial navigation is a three-dimensional process. For this reason system initialization requires externally-derived components of position and velocity along three non-collinear, out-of-plane directions or, in short, vector position and velocity.
- The basic inertial reference frame in all earth-originated missions is a set of inertially-fixed axes with origin at the center of the earth. Since the motion (rotation) of the earth relative to the set of axes is precisely known, initialization of an inertial system can be achieved through availability of vector position and velocity data relative to the earth's fixed surface.

Commonly Employed Initialization Options - Figure 2.3-14 depicts the major methods of inertial system initialization in current use. A basic separation of the procedures employed and equipment required for initialization is created by mission reaction time considerations; specifically, whether or not there is sufficient time to initialize the inertial system before the vehicle embarks on the mission. Initialization commences with power-up of the system and (usually) some form of coarse alignment. Coarse alignment is accomplished by comparison of the inertial system attitude outputs to the best

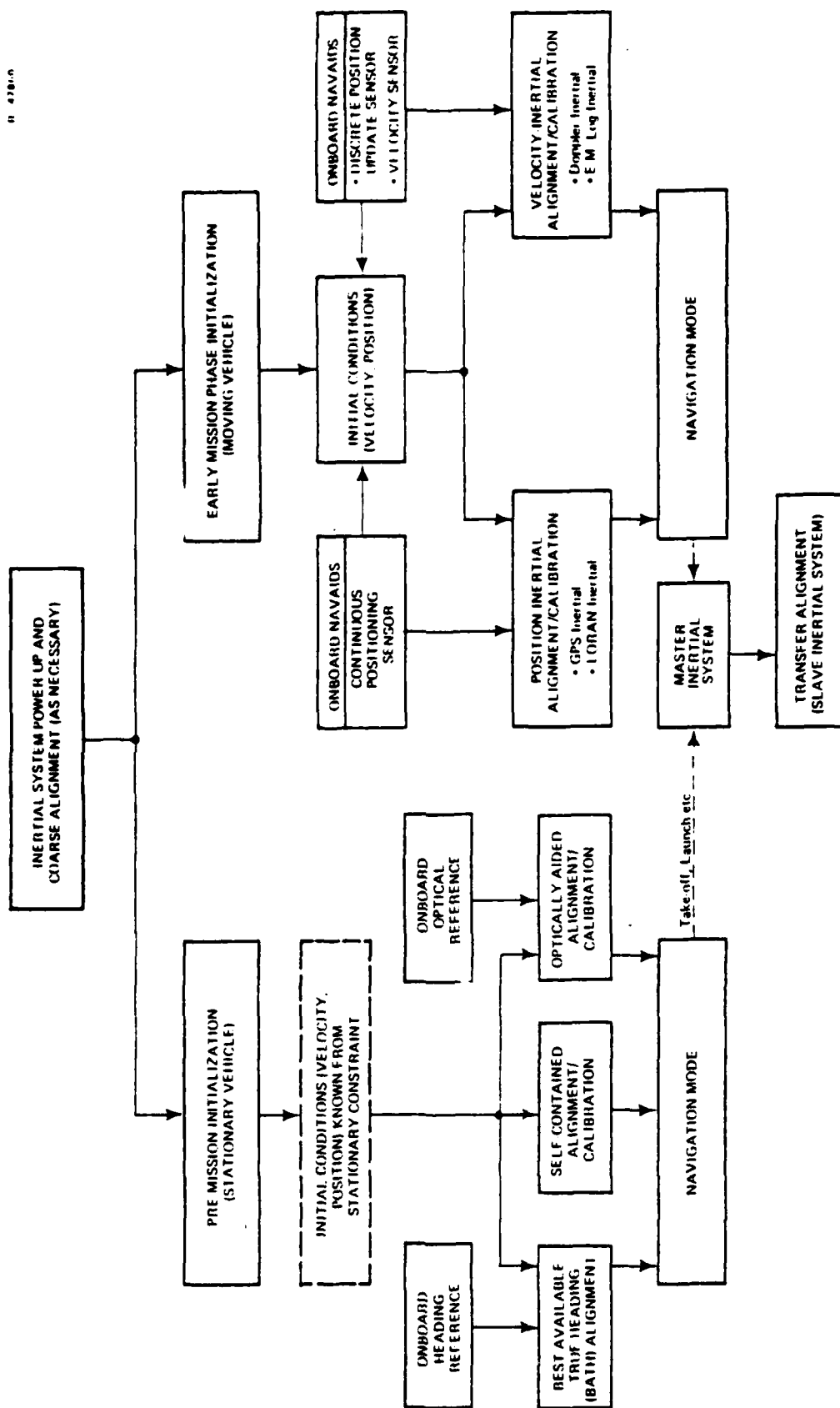


Figure 2.3-14 Inertial System Initialization Options

available external attitude references on the vehicle. In a parked aircraft, for example, coarse alignment may proceed on the assumptions that the airframe is roughly level and that the aircraft compass system provides an approximate indication of true heading. In inertial systems using mechanical (spinning rotor) gyros, special procedures may have to be followed to protect the gyros when the wheels are spun up. This is particularly true when coarse alignment involves externally imposed rotations of the structure carrying the gyros. In these cases the gyros are caged during coarse alignment by the use of electronic capture servos which prevent damaging motions of the internal elements of the gyros relative to the gyro cases.

#### Pre-Mission Initialization (Stationary Vehicle) -

When the vehicle is stationary relative to the surface of the earth, the initial values of position are usually known explicitly from prior survey (position of the launch site, dock, or parking apron). Initial velocity values are known implicitly from the stationarity constraint (ground speed and altitude rate are equal to zero). Moreover, to some degree of accuracy that depends on vehicle deviations from a truly motionless state (resulting from wind gusts acting on the aircraft, deflections of the missile suspension system in the silo, motions of the vessel allowed by the moorings, etc.), position and velocity of the vehicle can be determined over an extended period of time. These factors create sufficient conditions for the self-contained alignment of the inertial system. This process is described later in further detail.

The process of self-contained alignment suffers from two limitations which may be unacceptable in some operating scenarios:



- It is time-consuming, particularly in the solution for the azimuth alignment of the inertial sensors (orientation of the sensors in the local horizontal plane relative to a known geographical reference direction, e.g. true north)
- The final azimuth alignment solution accuracy is limited by a systematic error mechanism that may require employment of a distinct inertial system calibration mode for its correction. (Note that this calibration procedure can often be successfully conducted without the use of external references so that the composite alignment/calibration process is still self-contained.)

These limitations lead to the occasional use of external azimuth alignment data sources for more rapid and/or accurate inertial system initialization prior to a mission. When speed of reaction is the most important factor, a rapid azimuth solution of tolerable accuracy may often be achieved by comparison of the INS's azimuth orientation with other on-board directional sensors. This process is termed Best Available True Heading (BATH) Alignment. Because of errors associated with the other onboard directional sensors, the resulting initialization of the inertial system is usually less precise than that obtainable through a self-contained azimuth alignment process. In fact, the most commonly encountered BATH Alignment mechanization occurs on tactical aircraft and relies on spotting of the aircraft (no aircraft movement since shutdown after a prior mission). The last-recorded value of aircraft heading (from the inertial system) is stored in the navigational computer. This is one alternative to relying on on-board sensors.

In applications requiring very precise initial azimuth alignment (for example, ballistic missile guidance) the system designer frequently has recourse to the use of external alignment references that transfer accurately pre-surveyed angular information to the inertial system. Optical transfer devices have been used for this purpose, a typical combination being a photo-electric autocollimator (point light source, optical elements for creating a parallel beam of light, and a photosensitive detector to pick up the reflected light beam) external to the vehicle. The component of the system contained on the missile is a plane reflecting surface (or a 180 deg-reflecting prism) mounted on the structure carrying the inertial sensors. The method of determining the azimuth orientation of the optical transfer device varies widely with the operational scenario. In some cases it forms part of a portable Azimuth Laying Set (ALS) which gyroscopically determines the direction of local horizontal north. In others it is part of a theodolite assembly used to look at a distant reference point for azimuth determination.

In the foregoing discussion of initial azimuth alignment processes, it is assumed that the inertial system can be aligned relative to the local gravity vector (leveling) with sufficient accuracy to assure successful mission performance. The reasons behind this assumption are discussed in more detail in the section on INS alignment.

Early Mission Phase Initialization - The initialization of an inertial system on a moving vehicle requires other onboard sources of position and velocity data. Many types of equipment can provide these data (including another, pre-aligned inertial system). Several ingenious schemes involving constraints on the early path of the vehicle have also been used to provide implicit position and velocity data in some specific

applications. In most modern applications, however, satisfactory inertial system initialization in a moving vehicle requires either: (1) position sensors capable of providing near-continuous, accurate position information (from which vehicle velocity can be derived) or (2) a combination of discrete position updating sensors (radar, sextant, landmark range and bearing measuring devices, star-tracker, etc.) combined with continuous velocity sensors.

Typical examples of currently-used or planned continuous position sensors are LORAN and Global Positioning System (GPS) receivers. The latter provide the three-dimensional positional information required for complete inertial system initialization. The former require use of an altimeter or depth gauge on the vehicle to provide the vertical position (altitude) information needed for complete inertial system initialization. Radio navigation aids are discussed at greater length in Section 2.3.7.

Typical velocity sensors in contemporary use are Doppler radar and Doppler sonar systems. These can be mechanized to provide altitude (or depth) rate in addition to the components of ground speed lying in the horizontal plane. Frequently, however, the vertical velocity component is provided by other sensors. To a degree of accuracy determined by flight or cruise conditions, such devices are true groundspeed sensors. In marine vessels electromagnetic (e.m.) logs are used to provide velocity inputs for inertial system initialization purposes. These measure the speed of the vehicle relative to the water along its longitudinal axis and require estimates of any currents relative to the surface of the earth to achieve complete accuracy. On ground-based vehicles an accurate speedometer (and natural constraints on side-slip and vertical motion of the vehicle) usually provide very satisfactory velocity references for inertial system initialization.

The termination of the inertial system initialization process is often marked by a deliberate change of mechanization, signaling entry into the Inertial Navigation Mode. In many moving vehicle applications, however, it is advantageous to continue the use of external position and/or velocity data throughout the mission to enhance the overall navigation system capability. In either situation, at some point in time the inertial system can be described as being adequately initialized for mission purposes. It then becomes available as a Master Inertial System providing acceleration, velocity, position, and attitude outputs on a continuous basis. These outputs can be used for the initialization of other (slave) inertial systems on board the vehicle.

As a final, general observation on the initialization of inertial systems on moving vehicles, note that the externally-derived acceleration, velocity, or position data may be provided in two distinct forms:

- Components in an external reference frame that can be directly related to the inertial system's reference frame (for example, local north, east, and vertical)
- Components in the vehicle reference frame (roll, pitch, and yaw axes).

Continuous position measurement systems such as GPS and LORAN fall into the former category as do Master Inertial Systems. It is characteristic of the resulting alignment processes that they are greatly enhanced in speed and accuracy by accelerations of the vehicle. For cruise vehicles in particular (aircraft and surface vehicles), horizontal accelerations are very beneficial in speeding up the azimuth alignment process and are frequently programmed into the vehicle mission profile for this purpose (in the form of deliberate turns or weaving flight patterns).

Continuous velocity measurement systems such as Doppler radar, Doppler sonar, true airspeed sensors, and e.m. logs fall into the second category. This situation requires a transformation of the externally derived velocity data into coordinates that can be related directly to the inertial system's reference frame. To avoid the need to equip the vehicle with an onboard attitude reference of accuracy comparable to, or better than, the inertial system itself, this transformation is usually conducted using the (initially) coarsely-aligned inertial system to provide the data relating vehicle axes to inertial reference axes. The resulting mechanization is characterized by alignment processes with dynamics and solution accuracies that are largely independent of the vehicle motion.

Introduction to Alignment Principles<sup>(†)</sup> - The objective of inertial system alignment is the accurate orientation of the inertial sensors (gyros and accelerometers) relative to the chosen navigation axes, prior to beginning actual inertial navigation or guidance. The alignment process may take one of two forms:

- Estimation of the angular displacement between the inertial system platform and navigation coordinate frame
- Estimation and Control of the inertial system platform orientation, resulting in its alignment to a preferred orientation.

The former method is the only option for strapdown inertial systems (where the inertial system platform is fixed to the vehicle). It is also used in some non-strapdown applications (notably in ballistic missiles). In both cases the estimation

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(†) This section contains material at a more advanced level than the rest of the text.

process results in the storage of accurate values for a set of large angles in the navigational computer, usually in the form of direction cosines relating the inertial system platform to the navigation axes.

The Estimation and Control method is extensively used in gimballed inertial systems when performance gains can be achieved and/or desirable outputs easily obtained by accurately positioning the inertial sensor cluster to the preferred orientation.

The navigation axes chosen in a given application depend both on the mission and on the characteristics of the inertial system. However, in terrestrial applications, all forms of navigation axes have one thing in common -- they are, by definition, earth-related. That is, they all possess some definable geometrical relationship to a set of earth-fixed axes, either at the origin point of the mission, the position of the vehicle, or the destination (target). Thus alignment of an inertial system involves defining the orientation of the inertial system platform relative to an earth-related frame prior to entry into the navigation/guidance mode.

To achieve this alignment it is necessary to use earth-related physical phenomena that can be transferred to, or measured autonomously by, the inertial system in a manner that provides the desired relative orientation information. Two measurable physical quantities that fit this description are the gravity vector at any fixed location on the surface of the earth and the earth's rotation vector.

Figure 2.3-15 illustrates the relationship between these two quantities. Note that the gravity vector,  $g$ , provides a measurable reference force at any point on the surface

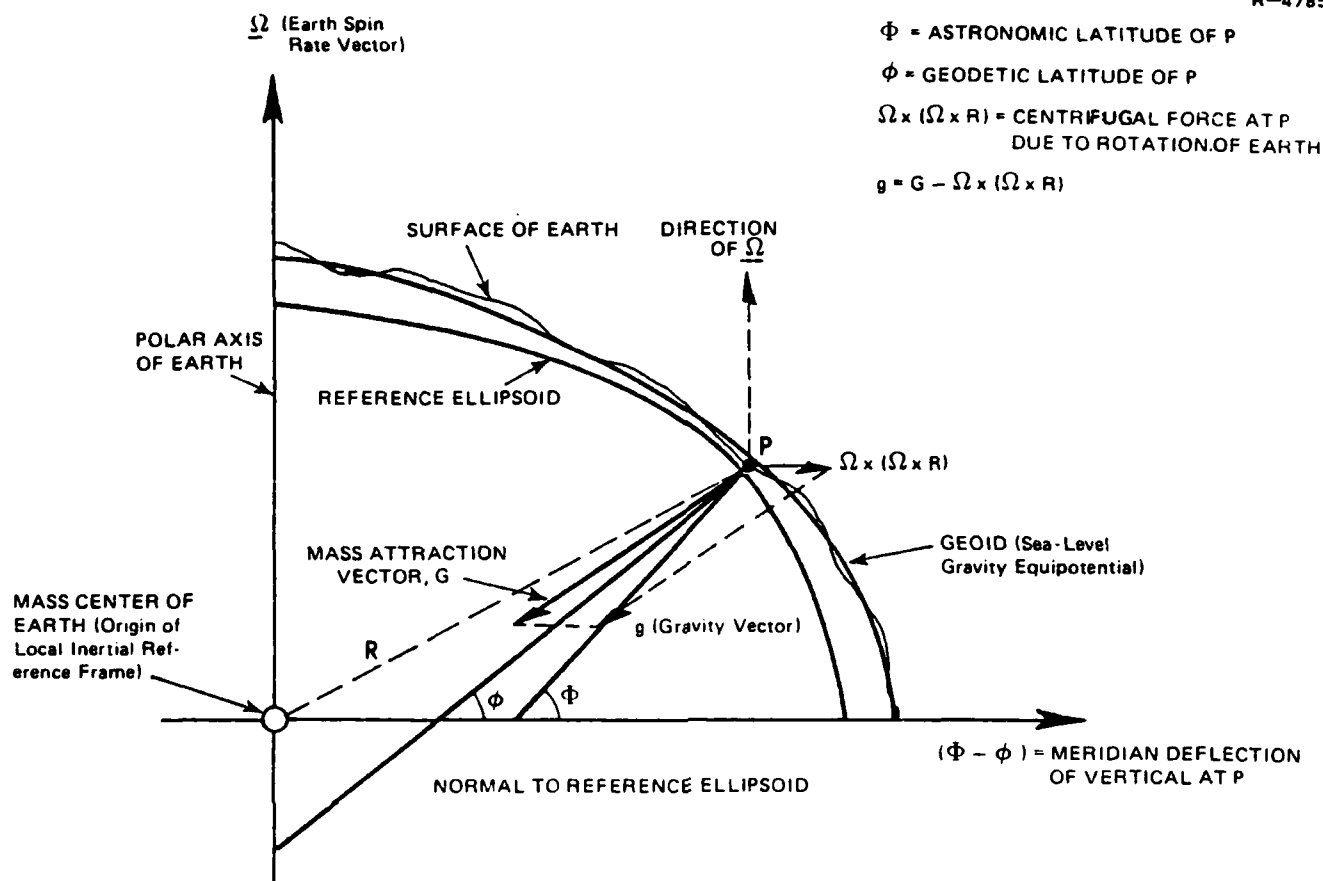


Figure 2.3-15 Gravity and Centrifugal Acceleration Vectors at Earth's Surface

of the earth. If this force is used as one of the physical quantities sensed for alignment, the inertial system will be aligned relative to the astronomic (plumb bob) vertical -- that is, to astronomic latitude and longitude. If the inertial system position initialization is now entered into the navigation computer in the more commonly available geodetic coordinates a circumstance exists which leads to the propagation of a small error. This error can be significant in some applications requiring great precision (e.g., ballistic missiles), and special corrective measures must be taken involving knowledge of launch-point deflections of the vertical.

Autonomous Inertial System Alignment (Stationary Vehicle) - It was pointed out previously in this section that, in vehicles that are stationary on the surface of the earth, self-contained alignment of the inertial system is possible. The basic concepts of this process are briefly described here. This alignment technique is widely used in pre-mission system initialization and the principles involved are employed (with suitable modifications) in moving vehicles.

Alignment Relative to the Gravity Vector (Leveling)<sup>(†)</sup> - When a vehicle is stationary on the surface of the earth, the only force acting on the vehicle and its contents is the reaction force exerted by the surface of the earth. This is equal in magnitude and opposite in direction to the force of gravity on the vehicle. In this situation, the outputs of ideal inertial system accelerometers are simply the normalized components of this single force along the directions of their sensing axes. This is illustrated in Fig. 2.3-16a. The measured outputs of the accelerometers in this situation are:

$$\begin{aligned} f_x &= f \cos (90^\circ - A_x) = f \sin A_x \\ f_y &= f \sin A_y \\ f_z &= f \sin A_z \end{aligned} \quad (2.3-22)$$

where, for a stationary vehicle, the net force vector is, simply,  $f = |g|$ .

The angles  $A_x$ ,  $A_y$ ,  $A_z$  define the inertial system platform orientation relative to the local level plane and can be estimated from the measured outputs of the accelerometers.

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(†) This section contains material at a more advanced level than the rest of the text.



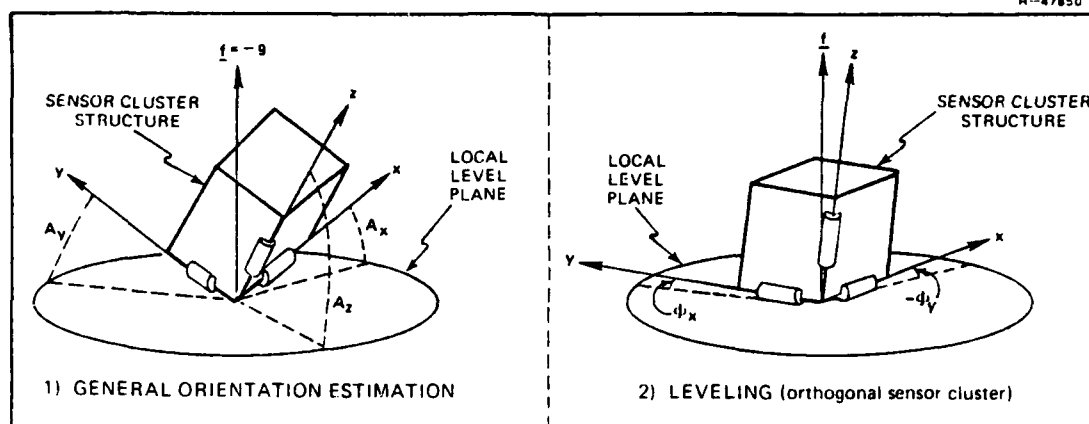


Figure 2.3-16 Autonomous Alignment Relative to the Gravity Vector

The relationships of Eq. (2.3-22) are used. This type of procedure is used for alignment relative to the gravity vector in all strapdown systems and in some gimballed systems (usually in ballistic missiles) where large values of the angles  $A_x$ ,  $A_y$ , and  $A_z$  characterize the preferred inertial system platform orientation. The local value of  $g$  needs to be known accurately for this procedure, although it may be computed internally from:

$$g = (f_x^2 + f_y^2 + f_z^2)^{1/2} \quad (2.3-23)$$

when  $x$ ,  $y$ , and  $z$  are mutually orthogonal, or an equivalent expression for non-orthogonally-mounted accelerometers.

Because of errors and noise sources inherent in the accelerometers and slight random vehicle movements, the estimation process requires a finite time (two to five minutes in

most applications) for the accurate determination of  $A_x$ ,  $A_y$ , and  $A_z$ . At the end of the process the values obtained for these angles are stored in the navigation computer.

In many applications it is required that a principal reference plane (e.g., the x-y plane) in the sensor cluster be physically aligned with the local horizontal plane. (Recall from Section 2.3.1 that the class of systems mechanized in this fashion is called local-level.) Such a system is initialized by bringing the inertial system platform into approximate alignment with the locally level plane by a mechanical adjustment with respect to known vehicle axes. Then  $A_x$  and  $A_y$  become tilt error angles ( $\phi_y$  and  $\phi_x$ ). The x and y accelerometer outputs that result are then used to drive the inertial system platform (through the gyros and gimbal stabilization servo loops) to the desired level state. The action ideally stops when the measurements of the accelerometer outputs ( $f_x$  and  $f_y$ ) reach zero. This process is commonly known as leveling. It is in widespread use in a variety of cruise vehicle inertial navigation systems (aircraft, ships, and ground transporters).

Alignment Relative to the Earth Spin Rate Vector  
(Azimuth Alignment)<sup>(†)</sup> - The previously described alignment processes do not determine the orientation of the inertial system platform about the gravity vector (subsequently referred to as Azimuth Alignment). In fact, they are not sufficient for the complete alignment of the inertial system platform relative to a triad of earth-fixed axes. This is a reflection of the general fact that the complete alignment of one three-axis coordinate frame with a three-axis reference

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(†) This section contains material at a more advanced level than the rest of the text.

frame requires the measurement of two non-collinear vectors in both frames (or, in the situation of current interest, the measurement in the inertial system platform frame of two non-collinear vectors whose magnitude and direction are known in the earth-reference frame).

Except in polar areas of the earth's surface, the direction of the earth's spin rate vector is not collinear with the local gravity vector (see Fig. 2.3-15); hence measurement of the effects of earth spin rate on the behavior of the inertial system can provide the means for azimuth alignment of the inertial system platform. In polar areas the complete, autonomous alignment of an inertial system is not practicable and externally-provided azimuth references are required.

The principles behind the azimuth alignment process are illustrated for local-level inertial systems that have already been leveled. Two situations of common interest are depicted in Fig. 2.3-17. In the first, the objective is to estimate the inertial system platform heading angle,  $A_H$ , and to store the final estimate (at the end of the alignment process) in the navigation computer.

From Fig. 2.3-17a it can be shown that the components of the earth spin rate vector about the inertial system platform's x and y axes are given by:

$$\begin{aligned}\Omega_x &= \Omega \cos \phi \sin A_H \\ \Omega_y &= \Omega \cos \phi \cos A_H\end{aligned}\tag{2.3-24}$$

If  $\Omega_x$  and  $\Omega_y$  can be measured by the inertial system, then the platform heading angle can be estimated from:

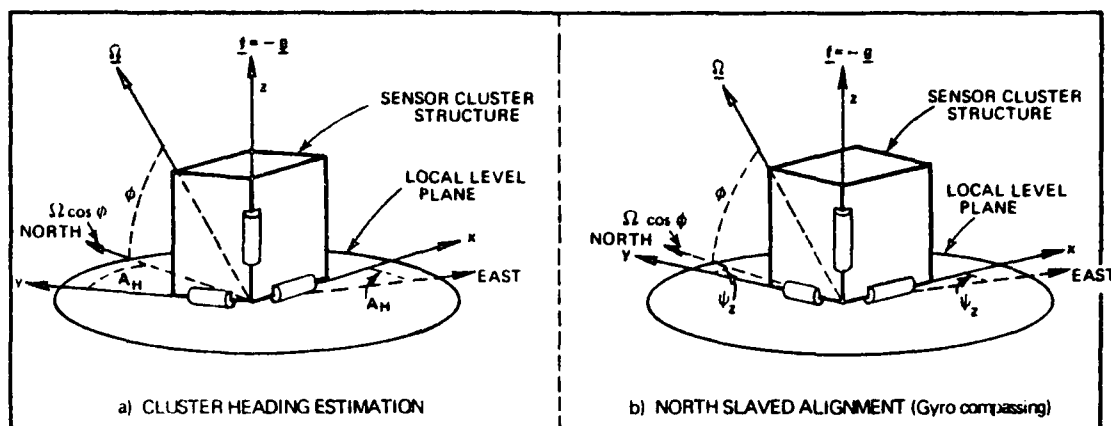


Figure 2.3-17 Autonomous Azimuth Alignment

$$A_H = \arctan \frac{\Omega_{xM}}{\Omega_{yM}} \quad (2.3-25)$$

where  $\Omega_{xM}$  and  $\Omega_{yM}$  are the measured components of earth spin rate.

The method of measuring  $\Omega_x$  and  $\Omega_y$  involves the inertial system accelerometers. The actual inertial system platform rotation rate relative to inertial space is determined by torquing rate commands  $\omega_x$ ,  $\omega_y$  and  $\omega_z$ , applied to the inertial system gyros from the navigation computer. The platform rotation rate relative to the earth (the direction of the local g vector) is defined by the time rates of change of the platform tilt angles  $\phi_x$ ,  $\phi_y$ , and  $\phi_z$  defined in Section 2.3.2 and illustrated (for  $\phi_x$  and  $\phi_y$ ) in Fig. 2.3-16. The tilt rates about x and y lead to accelerometer outputs,  $f_y$  and  $f_x$ , whose time rates of change are related to the tilt rates by:

$$\begin{aligned}\dot{f}_x &= -g \, d/dt(\sin \phi_y) \approx g\dot{\phi}_y \\ \dot{f}_y &= g \, d/dt(\sin \phi_x) \approx g\dot{\phi}_x\end{aligned}\tag{2.3-26}$$

Measurement of the rates of change,  $\dot{f}_x$  and  $\dot{f}_y$ , of the accelerometer outputs thus provides measured values  $\dot{\phi}_{yM}$  and  $\dot{\phi}_{xM}$  for the platform tilt rates from which the earth spin rates are computed via:

$$\begin{aligned}\Omega_{xM} &= \omega_x - \dot{\phi}_{xM} \\ \Omega_{yM} &= \omega_y - \dot{\phi}_{yM}\end{aligned}\tag{2.3-27}$$

Thus, the measured earth rate equals the inertial system platform's rotation rate relative to the inertial space, minus measured platform rotation rate relative to the earth. Equation (2.3-25) is then used to estimate  $A_H$ .

In North-Slaved local-level systems, as illustrated in Fig. 2.3-17b, the objective is to drive the inertial system platform's x and y axes to point east and north respectively, starting from an approximate east-north orientation attained through Coarse Alignment. In this case it can be shown that the essential information on the residual, small azimuth misalignment angle\*,  $\psi_z$ , appears only as a tilt rate about the x axis. Thus the output of the y accelerometer contains the only measurement needed to estimate  $\psi_z$ . The estimate of  $\psi_z$  is then used, via an appropriate control function, to derive a rotation rate command signal. This signal is applied to the z gyro (and gimbal stabilization servo system) to null out the azimuth misalignment. Gyrocompassing (the name given to this process) is complete when the tilt rate,  $\dot{\phi}_x$ , is reduced to zero.

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\*Recall definition of  $\psi$  from Section 2.3.2.

The current state of the art in inertial sensor (accelerometer and gyro) design usually leads to azimuth alignment processes which require more time than the leveling process. (Gyrocompassing times are typically 10 to 30 minutes.) However, since the continued existence of tilt rates  $\dot{\phi}_x$  and  $\dot{\phi}_y$  during gyrocompassing inevitably leads to non-zero values of  $\phi_x$  and  $\phi_y$ , re-leveling of the sensor cluster is necessary throughout the gyrocompassing process. It is therefore usual to find leveling and gyrocompassing conducted as simultaneous or interleaved operations in most system mechanizations.

Alignment in Moving Vehicles - Autonomous alignment of an inertial system is not possible to any degree of accuracy in a moving vehicle. This is because the additional accelerations and rotation rates caused by vehicle motion appear in the INS alignment equations presented in the previous sections. Note the reappearance of the same problem of being unable to distinguish between gravitation and kinematic acceleration (discussed earlier in Section 2.3.1, and in Sections 1.2.7 and 1.3.3 of Unit One). In this instance, vehicle horizontal accelerations change the apparent direction of the  $g$  vector while vehicle velocity adds additional rotation rates that corrupt the gyrocompassing process.

Nevertheless, alignment of an inertial system in a moving vehicle in a manner analogous to that described for stationary vehicles is possible, when an external velocity (groundspeed) measuring system is available. The external reference provides the additional information needed to solve for the extra unknown accelerations and rotation rates appearing in the alignment equations. To accomplish this an external reference system must provide velocity inputs that can be transformed into components either along geodetic axes (for example, local north, east, and vertical) or along the inertial system platform axes.

External reference data are often available in a geodetic reference frame (as in the case of velocity data derived from continuous positioning sensors such as GPS and LORAN receivers) or can be transformed into a geodetic frame using attitude references external to the inertial system. These mechanizations result in alignment processes that are affected (beneficially) by vehicle maneuvers and course changes. The term Spatial Rate Gyrocompassing is frequently used to describe one such alignment process.

The inertial system platform axis mechanization option relies on the (coarsely aligned) inertial system to provide the attitude data whereby velocity inputs are transformed from vehicle axes into inertial system platform axes. The resulting alignment processes are virtually insensitive to the vehicle flight profile. The term Earth Rate Gyrocompassing is often used to describe the azimuth alignment process portion of this mechanization option.

A detailed, quantitative description of external velocity references is beyond the scope of this course. In general, velocity navaids (navigation aids) are used to restore the inertial system's ability to measure the direction of local gravity and the value of earth-rate components in a moving vehicle. The more general topic of aided-inertial systems, which, as a special case, includes moving vehicle inertial system alignment, is touched upon in Section 2.3.7.

Geodesy and the Inertial System Initialization Process - The principles of autonomous inertial system alignment described in the previous section extend, in one mechanization form or another, throughout most current inertial system applications. Errors associated with the alignment processes tend to be dominated by inertial sensor and external reference data source

imperfections. In most applications the limited amount of filtering (estimation) time required to complete those processes is also a source of error. Thus, for example, accelerometer output bias errors (and fixed scale factor errors) lead to incorrect measurements of the magnitude of the  $g$  vector and its direction relative to inertial system platform axes. These result in platform orientation errors relative to the local vertical (tilts). In like manner, systematic time-varying errors in the accelerometer outputs together with fixed (bias) gyro drifts (spurious rotation rates due to internal gyro imperfections) interfere with the tilt rate measurement process. Since this process is central to azimuth alignment (gyrocompassing), final alignment accuracy is affected. This can be of serious consequence particularly in ballistic missile applications. Random, time-varying errors in the accelerometers, the gyros, the external velocity reference systems, or simply due to unknown vehicle motions at the pre-mission initialization site, place a lower limit on the time required for the system to separate out the small, systematic error propagation patterns attributable to misalignment. Noise-induced platform misalignments alone can lead to unacceptably long azimuth alignment times (for example, 10 to 30 min in cruise aircraft).

However, some specific applications dictate the use of very high precision (and very expensive) inertial sensors whose performance parameters (accelerometer bias, scale factor error, gyro drift bias, etc.) are very stable once the system is switched on and sufficient time is allowed for switch-on transients (mainly thermal effects) to die away. In these applications the pre-mission (or early mission) mode of operation is usually engineered to allow exploitation of the stability of the gyros and accelerometers through extensive performance parameter calibration routines embodied in the mission preparation process. The end result is an inertial system in



which the instrument errors are calibrated sufficiently well to achieve very high initialization accuracies. At these levels of accuracy, geodetic factors are significant in both the initialization and the calibration processes, as well as in the active mission phases.

The classical and almost exclusive example of this type of application is that of high accuracy, all-inertial ballistic missile guidance. However, some high precision, long-duration cruise vehicle applications are appearing that also may demand special consideration of geodetic factors in the alignment process. The discussions that follow are related to ballistic missiles and apply, as qualified in the text, to launch from fixed sites or from moving vehicles.

Ballistic Missile Guidance System Initialization Requirements - It is appropriate to restate the inertial guidance problem for long-range missiles in terms that relate to the initialization process. The basic requirements are:

- Both the launch point and the target point locations must be precisely defined in a navigation/guidance coordinate reference frame that is accurately maintained by the inertial system during flight
- The missile velocity at launch (i.e., at the end of the initialization process) relative to the navigation/guidance coordinate frame must be accurately known
- The inertial sensors must be calibrated to a performance level adequate to accomplish both initial alignment and subsequent mission accuracy requirements
- The inertial system platform must be accurately aligned relative to the navigation/ guidance reference frame.

Targeting methods in current use yield target position coordinates in terms of geodetic latitude, geodetic longitude, and target elevation above the reference ellipsoid (or similar parameters related to a worldwide geodetic system that can, in turn, be related to the reference ellipsoid). Navigation/guidance coordinate reference frames are also typically mechanized in terms of the reference ellipsoid. Similarly, the launch point coordinates are normally defined by geodetic coordinates. Thus, with the exception of sensor calibration, all of the initialization objectives (position, velocity, and attitude) are related to the reference ellipsoid. Some of the implications are discussed below.

#### Geodetic Requirements for Accurate ICBM INS Initialization

Alignment to the Level Plane - The leveling processes described earlier in this Section result in the alignment of the inertial system platform to the local vertical. Recall that the local vertical deviates from the normal to the reference ellipsoid by the vertical deflection angles  $\xi$  (North) and  $\eta$  (East). In the absence of other errors, the leveling process will therefore terminate with the inertial system platform tilted by angles of the order of 6 to 10  $\widehat{\text{sec}}$  from the local horizontal north and east axes, relative to the navigation coordinate reference frame (ellipsoid frame). For ICBMs, the resulting impact errors at the target can be significant (up to 30 m per  $\widehat{\text{sec}}$ ). Thus, it is of interest to know the values of those tilts and to compensate for them in the alignment process. To do this, measurement of the deflections of the vertical at the launch site (by astrogeodetic survey or other means) is required. During inertial system initialization the launch site deflection of the vertical data usually take the form of computer-stored values expressed as small fractions of the local value of gravity. These corrections are added to the outputs of the system accelerometers during alignment.

The inertial system platform then seeks the normal to the reference ellipsoid during the leveling process.

Azimuth Alignment<sup>(†)</sup> - Deflections of the vertical at the launch site also lead to azimuth alignment errors in the autonomous alignment process, unless steps are taken to correct for them. Recall that the autonomous azimuth alignment process requires the measurement of platform tilt rates,  $\dot{\phi}_x$  and  $\dot{\phi}_y$ , about the inertial system platform axes. The tilt rates are used (through Eq. 2.3-27) to derive measured values for the earth rate components,  $\Omega_x$  and  $\Omega_y$ , about those axes. The torquing command rates sent to the inertial platform from the navigation computer include terms representing earth spin rate components that are derived from geodetic latitude. These differ from the actual earth spin rates appearing along the platform axes which lie along or normal to the astronomic vertical. As a result, erroneous values for  $\dot{\phi}_x$  and  $\dot{\phi}_y$  are measured. The computation for  $\Omega_{xM}$  and  $\Omega_{yM}$  is likewise in error.

It can be shown through perturbation analysis of Eq. (2.3-27) that, for a north-slaved inertial system, the magnitude of azimuth error is given by:

$$\text{Azimuth error} = \eta \tan \phi \quad (2.3-28)$$

Thus, an east deflection of the vertical ( $\eta$ ) of 10 sec at a latitude of 50 deg leads to a 12 sec heading error. The resulting cross-track impact error for a nominal 6000 mile (9600 km) trajectory is of the order of 1/3 mile (500 m).

It has been previously mentioned that the autonomous azimuth alignment process can occupy too much time, particularly

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(†) This section contains material at a more advanced level than the rest of the text.

in modern strategic systems with short reaction time requirements. Gyro bias drift errors can be particularly troublesome by requiring very long times for adequate calibration. In these circumstances an external azimuth line-of-sight of high angular accuracy (of the order of one to five  $\widehat{\text{sec}}$ ) is required, together with a means for transferring it to the inertial system platform. This implies site preparation through a geodetic survey which uses star sightings to establish the direction of the polar axis. As stated earlier this information is typically utilized in a local optical transfer device at the launch site.

The azimuth alignment process (and subsequent navigation) relies on the constancy of the earth spin rate vector (both in direction and magnitude) relative to inertial space. Angular migrations of the polar axis relative to inertial space do occur (see Section 1.2.4 in Unit One), but their magnitude is small and usually neglected in current inertial systems.

Calibration<sup>(†)</sup> - It has been pointed out, in the earlier description of the estimation process for establishing the orientation of the inertial system platform relative to the gravity vector, that the numerical value of gravity ( $g$ ) at the launch site is required. This can be obtained from the accelerometer output readings. However, the existence of bias and scale factor errors in the accelerometers leads to incorrect inertial system platform orientation estimates if this procedure is used. Also, scale factor errors, both linear ( $\text{micro-}g^*$  per  $g$ ) and quadratic ( $\text{micro-}g^*$  per  $g^2$ ) are important in subsequent powered flight. Thus, it is desirable to have a

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(†) This section contains material at a more advanced level than the rest of the text.

\*Note: 1  $g$  = 980 gal (nominal surface value of gravity) and 1  $\text{micro-}g$  = 0.980 milligal. These units are often used in discussions of inertial systems.

means for calibrating the accelerometers during pre-mission operations. For modern ICBMs this operation involves rotating the inertial platform to a number of different fixed nominal positions relative to the g-vector. A launch site measurement of the absolute value of gravity (see Section 1.3.3 in Unit One) is also required. In some very precise ICBM applications, the absolute value of gravity is required to  $\pm 0.5$  micro-g. For some performance verification tests of such systems or their components, even higher accuracy in the knowledge of absolute gravity at the test site is required. Even the effect of the lunar gravity field must be taken into account in such applications.

Initialization in Moving Vehicles - Elaborate gravimetric surveys are generally impractical for moving missile launch platforms. Accurate system initialization in such systems must rely heavily on position and velocity data from external measurements or aids. These aids, in addition to providing the initial position and velocity conditions for missile launch, must be precise enough to allow accurate determination of the orientation of the inertial system platform relative to the navigation coordinate frame. This implies a capability to determine the launch point deflections of the vertical from a comparison of the inertial system outputs with the external sensor outputs. Some degradation of initialization accuracy is to be expected under these conditions.

#### 2.3.6 Effect of Launch Point Errors on Strategic Weapons Systems(†)

With the World Geodetic System defined in Section 2.2 and its application to target positioning and weapon system

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(†) This section contains material at a more advanced level than the rest of the text.

launch point positioning appropriately described, the effects of error in both the target and launch point positions may now be discussed. The discussion is presented in the context of the inertial navigation problem, primarily because of the realities that surround the current generation of U.S. weapon systems in which enroute navigational accuracy is a critical performance issue. Other contexts are also possible, and are addressed briefly at the end of this section. But for both current and emerging generations of USAF and USN strategic ballistic missile systems, as well as the current generation of USAF strategic cruise missiles, the inertial navigation system is the primary mechanism through which launch point and target position errors affect weapon system accuracy.

A detailed treatment of the theory of inertial navigation is outside the scope of this course, but a brief treatment of the principles and equations involved provides a useful framework with which to discuss the launch point and target geodetic issue. The following discussion might be viewed as an extension of the material introduced earlier in Section 2.3, particularized initially to the ballistic missile case.

As discussed in Unit One, the earth's gravitational field is a conservative field, and the gravitationally-induced acceleration of a vehicle located at any spatial point in this field is a function only of the vector position of the vehicle

$$\underline{g} = g(\underline{r}) \quad (2.3-29)$$

At a point  $P(\underline{r})$  in space above the surface of the earth, there exists a required velocity,  $\underline{V}_{REQ}^*$ , which is a function of  $\underline{r}$  and the position of the target, such that if the vehicle were to

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\*It is also a function of the desired trajectory altitude at apogee, or (alternatively) the reentry angle. Minor non-ideal effects (primarily atmospheric drag during reentry) are also taken into account in the calculation of  $\underline{V}_{REQ}$ .

acquire this velocity, it would travel solely under the influence of the gravitational field (with no external forces required) along a trajectory that would cause it to impact on the target. The calculation of  $\underline{V}_{REQ}$  as a function of  $\underline{r}$  and the target position is the guidance problem. There remains the navigation problem, which involves the continuous determination of values of position,  $P(\underline{r})$ , and velocity,  $\underline{V}$ , while the missile is enroute to the target, in order to provide a basis for steering the missile to reach  $\underline{V}_{REQ}$ .

The inertial navigation system measures the non-gravitational portion of the missile acceleration. This non-gravitational element is commonly called the sensed acceleration,  $\underline{a}_s$ . As discussed in Section 2.3.3, the total acceleration\* at any time is the sum of  $\underline{a}_s$  and the gravitational acceleration,  $\underline{g}$

$$\underline{a}_T = \underline{a}_s + \underline{g} \quad (2.3-30)$$

The gravitational acceleration cannot be measured by the inertial system, but since it is a function only of position, and the navigation system is keeping track of position, it can be calculated in the navigation computer. With the total acceleration thus available through a combination of measurements and calculations, the missile position and velocity can be determined continuously by integration

$$\underline{V}(t) = \int_{t_0}^t \underline{a}_T dt + \underline{V}_0 \quad (2.3-31a)$$

$$\underline{P}(t) = \int_{t_0}^t \underline{V}(t) dt + \underline{P}_0 \quad (2.3-31b)$$

---

\*That is, the second derivative of position.

where  $t_0$  is the time at which the inertial navigation process is initiated (normally just before launch), and  $\underline{P}_0$  and  $\underline{V}_0$  are the initial position and velocity at this time.

Recall that an inertial navigation system must be initialized in position and velocity before the navigation process begins. The INS subsequently calculates the cumulative change in position and velocity during the flight, by measuring and processing the non-gravitational acceleration time history of the vehicle. This view of the inertial navigation system as an open-loop spatial integrator is critical to the following discussion of launch point position error propagation.

It should be apparent at this point that knowledge of the relative locations and velocities of the launch site and the target site in a well-defined, common geodetic coordinate system is a necessary prerequisite to the navigation<sup>\*</sup> and targeting<sup>\*\*</sup> process for inertially guided weapon systems. Errors in knowledge of these quantities, and their effects on weapon system accuracy, are therefore of interest. In the following paragraphs, launch site and target site error propagation effects are addressed.

The error equations, which describe the propagation of error quantities through the inertial navigation system under the assumption that the errors are in the linear range (i.e., that the position errors are small compared to the radius of the earth, that the errors in the missile's velocity

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\*Determination, from the inertial system measurements, of the vehicle position and velocity during the powered portion of mission.

\*\*Determination of the aim point, translated into  $\underline{V}_{REQ}$  in the case of ballistic missiles.



are small compared to its actual velocity, etc.), are obtained by perturbation of the missile equations of motion (Eqs. 2.3-31a and 2.3-31b)

$$\delta \underline{V}(t) = \int_{t_0}^t \delta \underline{a}(t) dt + \delta \underline{V}_0 \quad (2.3-32)$$

$$\delta \underline{P}(t) = \int_{t_0}^t \delta \underline{V}(t) dt + \delta \underline{P}_0 \quad (2.3-33)$$

where

$\delta \underline{P}(t)$  = missile position error at time  $t$

$\delta \underline{V}(t)$  = missile velocity error at time  $t$

$\delta \underline{P}_0$  = initial position error

$\delta \underline{V}_0$  = initial velocity error

$\delta \underline{a}(t)$  = acceleration error

Errors in the launch point position are accordingly introduced as initial condition errors in the linearized error equations. Their propagation through the navigation process, however, is complicated by their continuing effect on the computed acceleration.

Recall from Eq. (2.3-30) that the gravitational contributions to the total acceleration of the vehicle must be calculated from the available position estimate. The acceleration error,  $\delta \underline{a}(t)$ , in Eq. (2.3-32) accordingly takes the expanded form

$$\delta \underline{a} = \delta \underline{a}_s + \delta \underline{g} \quad (2.3-34)$$

$$= \delta \underline{a}_s + \frac{\partial \underline{g}}{\partial \underline{P}} \delta \underline{P} \quad (2.3-35)$$

The term  $\delta \underline{a}_s$  in the above equation is the error in the measurement of non-gravitational acceleration. It is due primarily to navigation hardware imperfections, and is not of interest in this context.\* The second term in Eq. (2.3-35), however, shows how a position error, arising either from an initialization error or from any other source, feeds back into the navigation process through the gravity computation to appear as an acceleration error. The acceleration error subsequently integrates into further position errors. This is the well-known gravity feedback effect, which tends to dominate the short-term dynamics of inertial navigation systems in so many situations of practical interest.\*\* It is a consequence of the physical laws mechanized in inertial navigation systems and cannot be eliminated by design.

The response of the navigation system to errors in the initial conditions ( $\delta \underline{P}_0$  and  $\delta \underline{V}_0$ ) may be evaluated by expressing the gravitational acceleration specifically in terms of the vehicle position

$$\underline{g} = \underline{g}(\underline{r}) = - \frac{GM}{r^3} \underline{r} \quad (2.3-36)$$

where

$\underline{r}$  = vehicle position with respect to the center of the earth

GM = product of the universal constant of gravitation (G) and the mass of the earth (M)

r = magnitude of the vector  $\underline{r}$

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\*There is another component of  $\delta \underline{g}$ , not shown specifically in Eq. (2.3-35), which represents the error in modeling the gravity field. This term is discussed in Section 2.3.2. The second term in Eq. (2.3-35) represents the error in the computed gravitational accelerations caused solely by the error in estimated position.

\*\*This is another view of the Schuler dynamics discussed in Section 2.3.2.

and calculating the spatial gradient matrix  $\frac{\partial \underline{g}}{\partial \underline{p}}$  in Eq. (2.3-35). The position and velocity error profiles resulting from errors in launch point position and velocity are then determined directly from the linearized error equations (Eq. 2.3-32 and 2.3-33).

The details of the calculations are outside the scope of this discussion, but the results may be summarized as follows:

- Errors in the vertical component of the launch site position, and in the horizontal downrange component of the launch site velocity, lead to downrange position errors which increase with time as the missile proceeds toward the target
- All other components of position and velocity errors at the launch site lead to undamped oscillations in the horizontal components of subsequent navigation errors, varying with a period of slightly under 90 minutes. These are the well-known Schuler oscillations, illustrated\* in Fig. 2.3-18.
- There is a fundamental instability in the vertical channel of the navigation system. If a vertical position error appears, from any source at all, it increases exponentially with time.

In all three cases, the target miss sensitivities tend to be strong functions of the trajectory shape.

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\*Figure 2.3-18 illustrates the crossrange position and velocity error profiles that result from a launch site crossrange position error of one meter. The amplitude and the Schuler oscillation period are established primarily by the geophysical characteristics of the earth and only secondarily by the missile trajectory characteristics. The trajectory, however, determines the flight time, and therefore controls the magnitude of the navigation errors at the time of target encounter.

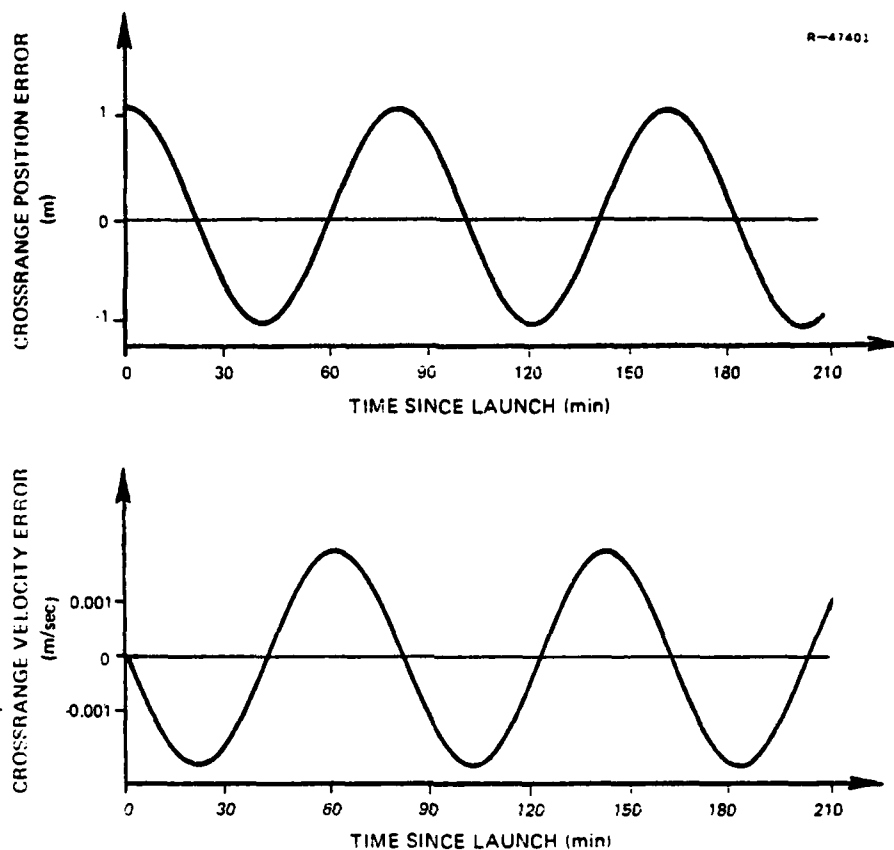


Figure 2.3-18 Crossrange Position and Velocity Error Profiles Resulting from Initial Crossrange Position Error

For the current generation of USAF ICBMs of the Minuteman class, the miss sensitivities to errors in launch site geodetics for a typical operational mission (North-firing, 5000 nm (9300 km) range, 600 nm (1100 km) apogee altitude, detonation on impact) are as shown in Table 2.3-1. The table lists downrange and crossrange miss contributions (in m) arising from errors of unit magnitude (1.0 m for position, 1.0 m/sec for velocity) in the position and velocity of the launch site. The miss sensitivities to the horizontal components of launch site position error are seen to be of the order of unity, with somewhat larger sensitivities for the launch site vertical

TABLE 2.3-1  
MISS SENSITIVITIES DUE TO LAUNCH POINT ERRORS

	MISS SENSITIVITY	
	DOWNRANGE	CROSSRANGE
<u>Position Error</u>	(Note 1)	
Downrange	-0.3	-0.4
Crossrange	0.1	0.6
Vertical	4.2	-1.2
<u>Velocity Error</u>	(Note 2)	
Downrange	4400	-480
Crossrange	100	-900
Vertical	1500	-500

Note 1: These quantities have no units  
(meters per meter)

Note 2: These quantities have units of seconds  
(meters per meter/sec).

position error. While actual error magnitudes for currently-deployed weapon systems are classified, it is well known that modern direct-survey techniques can locate any point in the Continental United States (CONUS) with respect to a central CONUS datum to within less than 15 m horizontally and 5 m vertically. Weapon system miss contributions due to launch site position errors at the one-sigma level are therefore likely to be well under 30 m.

The launch point velocity error miss sensitivities shown in Table 2.3-1 are normally not of interest for fixed-point ground-launched ballistic missiles.\* The magnitudes of

these sensitivities are seen to be extremely high, however, and the implications for mobile-based ballistic missiles as well as long-range air-launched cruise missiles are important.

In the case of missiles launched from a moving base (submarine, aircraft, etc.), the initialization of the missile navigation system must be carried out by transfer from the navigation system of the base vehicle. The only long-range U.S. ballistic missiles currently deployed which fall into this category are the SLBMs of the USN Poseidon type, although future force components could include USN Trident missiles and possibly air-launched versions of the USAF MX ICBM as well. The magnitude of the miss sensitivities to initial velocity errors is a strong function of the range to the target, but it is apparent from the example in the table that missile carrier velocity errors of a small fraction of a meter per second are required if CEPs below 500 m are of interest.

The error propagation equations and missile system initialization requirements discussed above apply equally to ICBMs and to inertially guided cruise missiles, with two additional considerations in the cruise missile case:

- For long-range vehicles of the Air Launched Cruise Missile (ALCM) class, the extended duration of the powered flight phase of

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\*The prelaunch velocity of ground-launched ballistic missiles is not zero. In an inertial (non-rotating) coordinate system, earth rotation effects produces launch site velocities of the order of 200 m/sec at moderate latitudes. The launch site velocity error, however, is normally very small. If the launcher (silo, etc.) is normally motionless with respect to the earth, the initial velocity error is due primarily to the initial position error, and can be computed from the earth's angular velocity. At a latitude of 45 deg, a worst-case (North) position error of 30 m produces a worst-case (East) velocity error of well under 0.003 m/sec.

the mission (several hours, as opposed to a few minutes with ballistic missiles) greatly magnifies the problem of instability in the vertical channel. Unaided inertial navigation in the vertical direction for this period of time is not practical, and an altimeter must be provided as a basis for (frequent) periodic updates to the vertical position and velocity estimates.

- For further practical reasons associated with the accuracy of current-generation strategic aircraft navigation systems and the expected accuracy of the ALCM-class missile navigation hardware, all-inertial ALCM navigation will not provide the accuracies that DoD requires for the anticipated ALCM missions. (This is not due to a fundamental limit on the accuracy of the inertial navigation process. It is primarily a reflection of inertial hardware costs.) In this situation, the problems associated with accurate cruise missile inertial navigation can be greatly mitigated by introducing a capability for occasional position updates, or "fixes," during the flight to the target. The net effect is a major reduction in the miss sensitivity to inertial system initialization errors.

Finally, it is appropriate to mention the effect of target position errors. These errors are introduced into the inertial guidance process as errors in the calculation of the required cut-off velocity\* which will result in impact on the target. The miss sensitivity to the horizontal components of the target location error is unity. But the sensitivity to a

---

\* $V_{REQ}$ , as defined after Eq. 2.3-29.

target height error is somewhat larger\* because of the target approach geometry (Fig. 2.3-19). The magnitude of the sensitivity depends on the trajectory parameters and the terrain characteristics in the target area, and typically lies between 2.0 and 4.0 (meters per meter).

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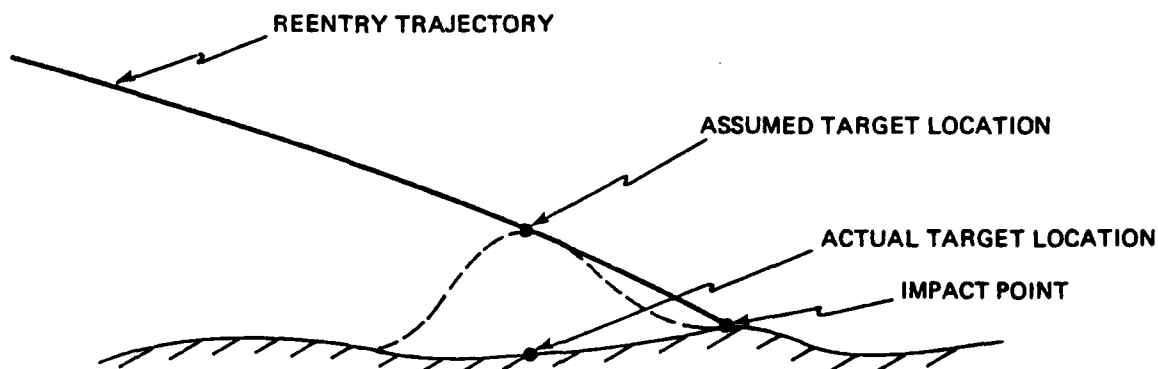


Figure 2.3-19 Propagation of Target Altitude Error into Downrange Miss

### 2.3.7 Aided Inertial Navigation Techniques

As has already been discussed, the errors of an unaided INS typically grow with time in an unbounded manner. As a consequence, the cruise inertial navigation systems used in modern aircraft, missiles, and submarines are normally aided or damped with data from external (i.e., non-inertial) sources, such as:

- Altimeter or depth gauge
- Speed reference (Doppler radar or ship's velocity log)
- Position reference (radio navigation system, navigation satellites, etc.).

\*With the warhead fuzed for detonation on impact.



The errors of an aided INS are typically bounded, although they grow between position fixes.

The techniques that are used to combine external reference data with INS outputs fall into two categories: "conventional continuous-feedback damping" and "Kalman-filter<sup>\*</sup> damping." Conventional continuous-feedback techniques are often used with altitude and speed reference devices, whereas Kalman-filter techniques are frequently used with position reference devices. However, the trend in recent years has been toward more extensive use of Kalman-filter techniques. An overall conceptual diagram of a multisensor-aided INS is shown in Fig. 2.3-20. A conceptual diagram of INS error compensation and correction is presented in Fig. 2.3-21.

Inertial system sensor errors may be divided into two categories:

- Errors associated with the measurement device itself are referred to as instrument errors
- Errors induced by external factors which operate independently of instrument quality are called environmental errors.

Examples of instrument errors are gyro drift, accelerometer bias, or radio receiver noise. Quantities such as deflections of the vertical, radio-wave propagation anomalies, and barometric variations due to weather patterns are in the environmental error category.

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\*Kalman filters are signal mixing and estimation algorithms designed to take advantage of the known statistical properties of system measurements and the errors that corrupt them.

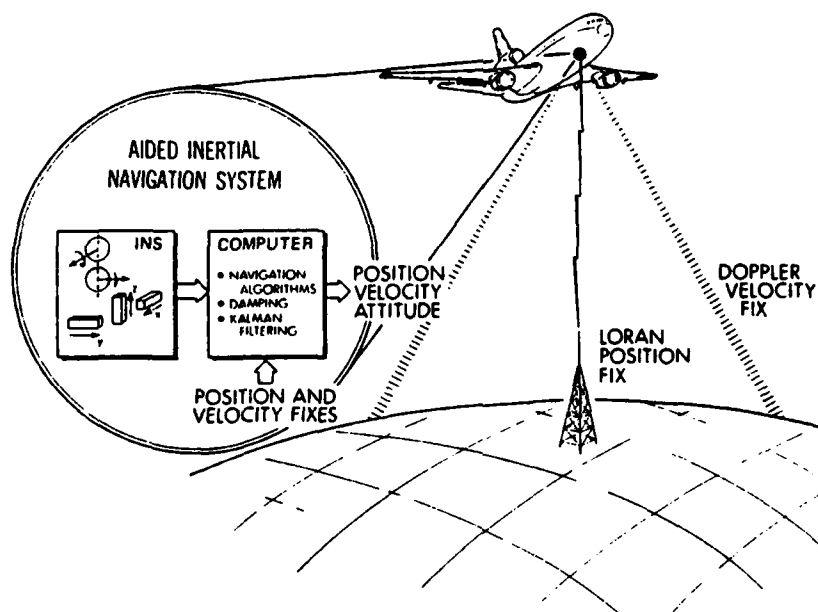


Figure 2.3-20 Multisensor Aided Inertial Navigation System Operation

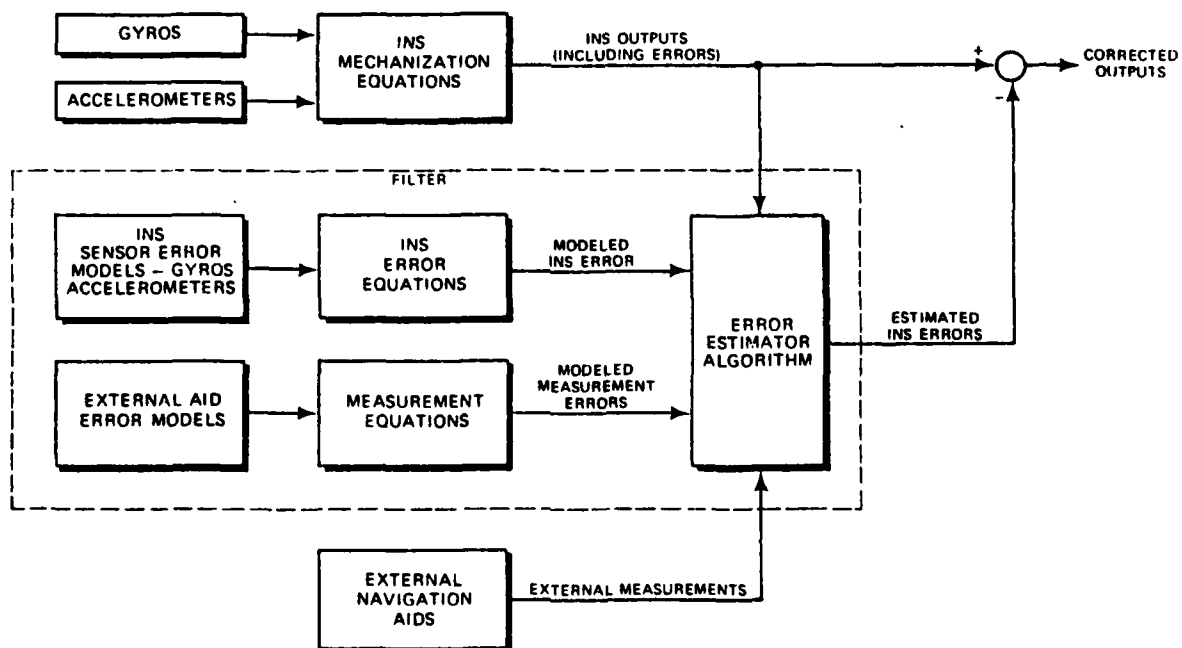


Figure 2.3-21 INS Error Estimation and Correction

Example of an External Aid (Altimeter) - Often an altimeter is used to stabilize the INS vertical channel.\* One simple configuration which is sometimes mechanized involves a filter to process the altimeter signal, as shown in Fig. 2.3-22. In this instance, the estimated altitude error is referenced to the accelerometer outputs and removed by subtraction. This corrects the velocity error as well as the altitude error.

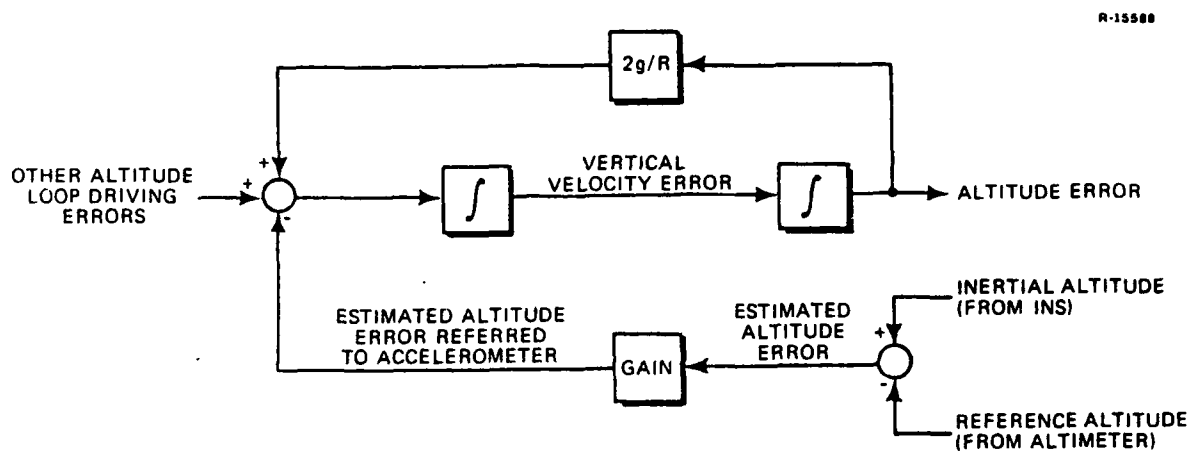


Figure 2.3-22 Static Stabilization of Altitude Error

Often simple filters are used in specific segments of the system\*\* (as in the previous example of vertical channel static stabilization), while a more powerful algorithm such as a Kalman filter is employed to estimate errors of the system as a whole. In such instances the local filter relations become incorporated into the appropriate INS error propagation equations or instrument models. The Kalman filter estimation

\*Recall from Section 2.3.2 that the INS vertical channel is unstable.

\*\*Instruments will frequently employ prefilters, which operate on the instrument output to provide a signal suitable for further processing (demodulation, spike removal, etc.).

algorithm then takes into account all of these relations in a system model and estimates errors on the basis of all known dynamics.

Stellar Inertial Guidance - A star sensor measurement can be used to provide an inertial system update by comparing the predicted star location with that actually observed. Since star locations are known to accuracies better than one  $\widehat{\text{sec}}$ , a stellar update can be used for the accurate reset of certain INS errors. This is accomplished as follows.

Prior to the mission a star is selected, which is to be observed at a particular time. Using an onboard clock and the star's celestial coordinates, the star sensor is pointed by the INS so that, at the correct time, the star lies in the center of the tracking instrument's field of view. However, because the inertial system platform typically possesses some error in its orientation, the star image is displaced from the center of the viewing field, as illustrated (for one type of star tracker) in Fig. 2.2-23. The INS platform orientation corrections required to center the star in the field are the measurements provided by the sensor.

In terms of the INS error quantities discussed in Section 2.3.2, the star sensor measures the two components of the computer platform misalignment angle ( $\psi$ ) that are perpendicular to the line of sight (LOS) to the star. That only two degrees of freedom can be measured by a single star sighting is apparent from Fig. 2.3-23. When the star image is at the center of the screen (no error indicated), the platform could still have an arbitrary amount of angular error about the LOS axis.

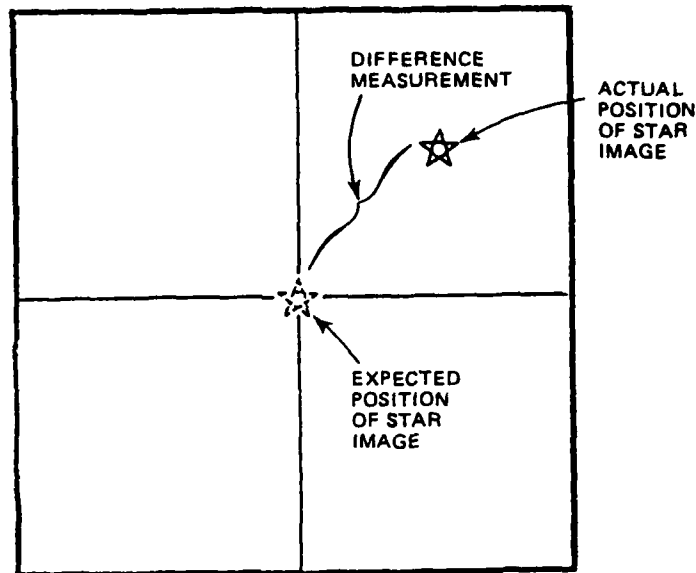


Figure 2.3-23 Star Sensor Measurement

It is appropriate to discuss why a star sighting often does not reduce position errors as much as might be hoped. Recall from Section 2.3.2 that the computer platform error  $\underline{\psi}$  is a combination of the actual platform tilt error,  $\underline{\phi}$ , and the tilt error resulting from position error,  $\underline{\theta}$ . Figure 2.3-24 illustrates the effect of a star measurement which eliminates  $\underline{\psi}$  angle errors. Note that Fig. 2.3-24 indicates that current position errors are unaffected by the measurement. In actual fact it turns out that some portion of position error is correlated with the observable components of  $\underline{\psi}$  angle error and can be estimated using advanced signal processing techniques. However, position and velocity errors originating from sources that are independent of  $\underline{\psi}$  are not observable by star sighting measurements. Thus, the effects of gravity disturbance errors or any other Schuler loop error source that is uncorrelated with  $\underline{\psi}$  cannot be improved by stellar guidance techniques.

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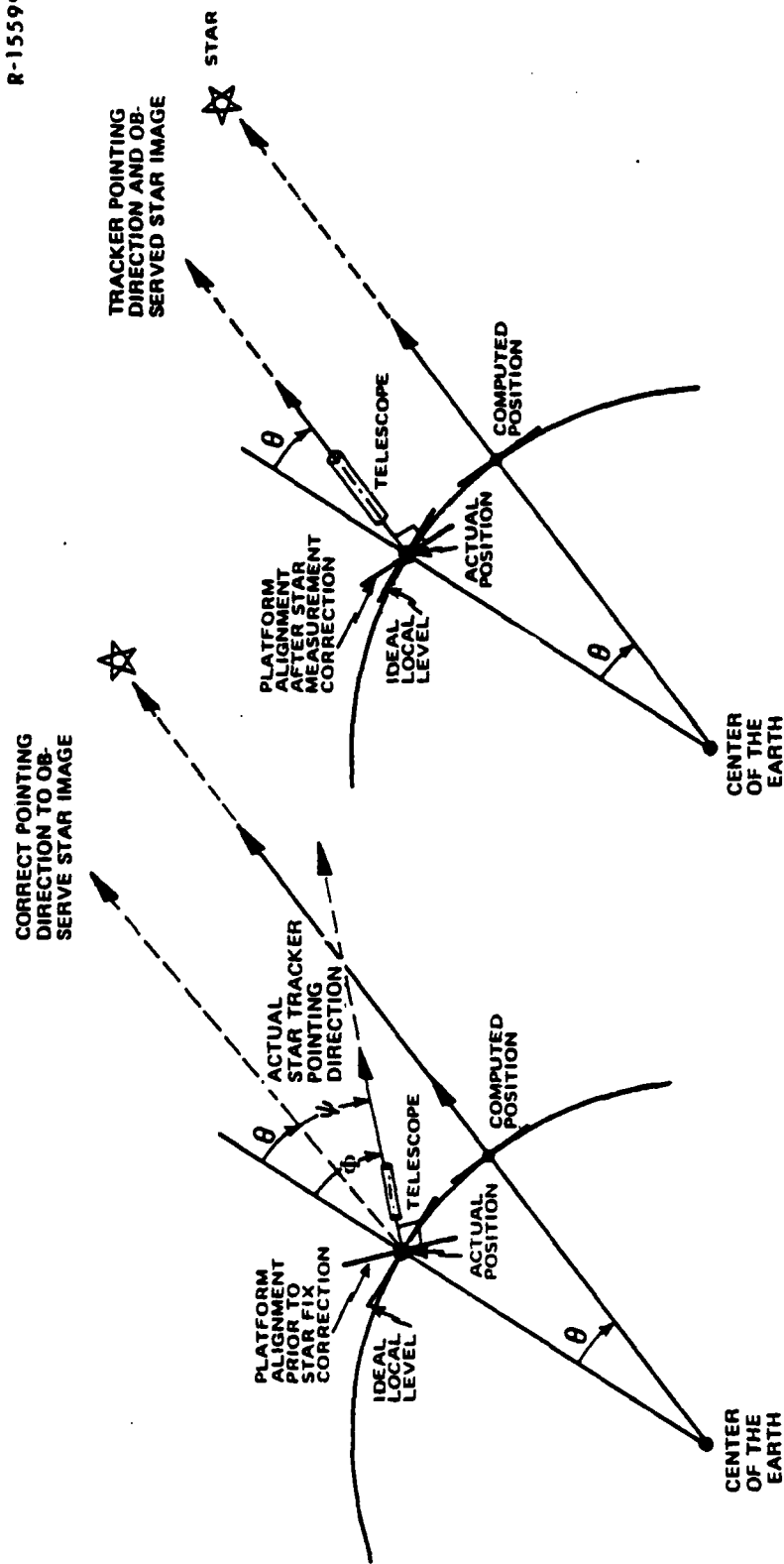
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a) BEFORE CORRECTION

b) AFTER CORRECTION

Figure 2.3-24 Star Tracker Measurement Illustrated for Local-Level Coordination of Error Angles

In the foregoing discussion, the conceptualization is used of a star tracker which nulls out the difference between the actual and computed position of the star in the viewing field. In practice, with ICBMs, a star fix can be implemented somewhat differently. Instead of physically slewing the inertial guidance platform, the misregistration of the star image from its computed screen position is stored in the computer. The corresponding  $\psi$  angle corrections computed from the misregistration distance are taken into account in subsequent inertial system calculations. In addition, because the boost portion of the trajectory is finished, the velocity error computed from the star fix is corrected by activating small thrust-er rockets on the warhead-carrying stage.\* The measure-compute-thrust, compute-thrust, etc. sequence is illustrated in Fig. 2.3-25.

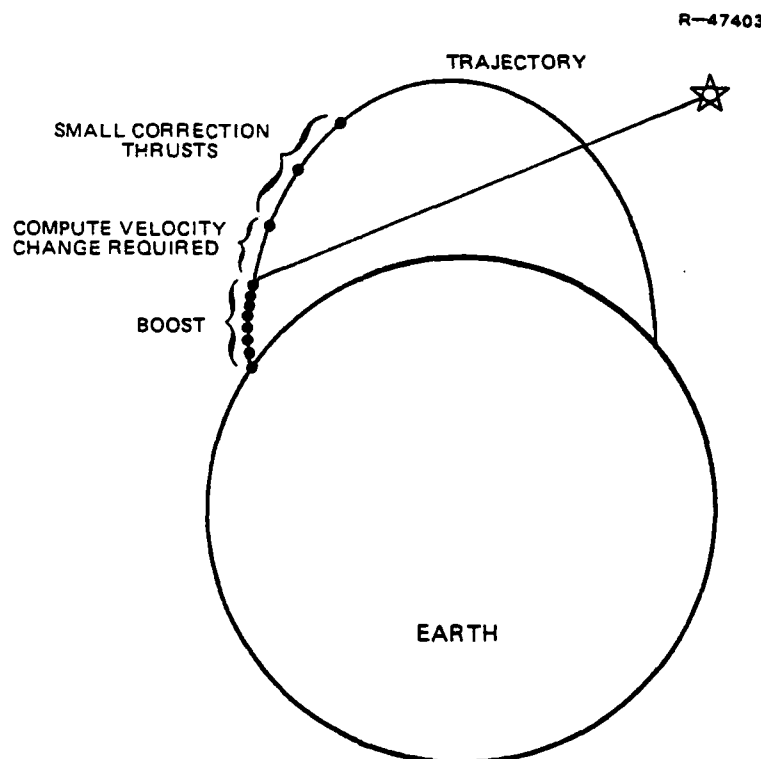


Figure 2.3-25 Implementation of Stellar Update

\*Often referred to as the "bus."



A stellar observation is accomplished somewhat differently on a missile stage which is spinning. On such a vehicle the star sensor is aimed so that, at a certain instant, the star image nominally crosses the center of the sensor. The sensor's screen consists of several diametral photosensitive slits as illustrated in Fig. 2.3-26. As the vehicle turns, the star image passes over each slit and generates sequential event times. The time differences between these observations and their collective variation from the expected time of the star sighting are used to calculate the angular corrections required to center the star in the sensor's viewing field. Note from Fig. 2.3-26 that the further off-center the star image falls, the greater is the time lag between events. For the case in which the sensor begins the viewing sequence in perfect alignment, the star image track passes through the center of the field and only one event is observed, which synchronizes exactly with the precomputed time at which the star observation is expected.

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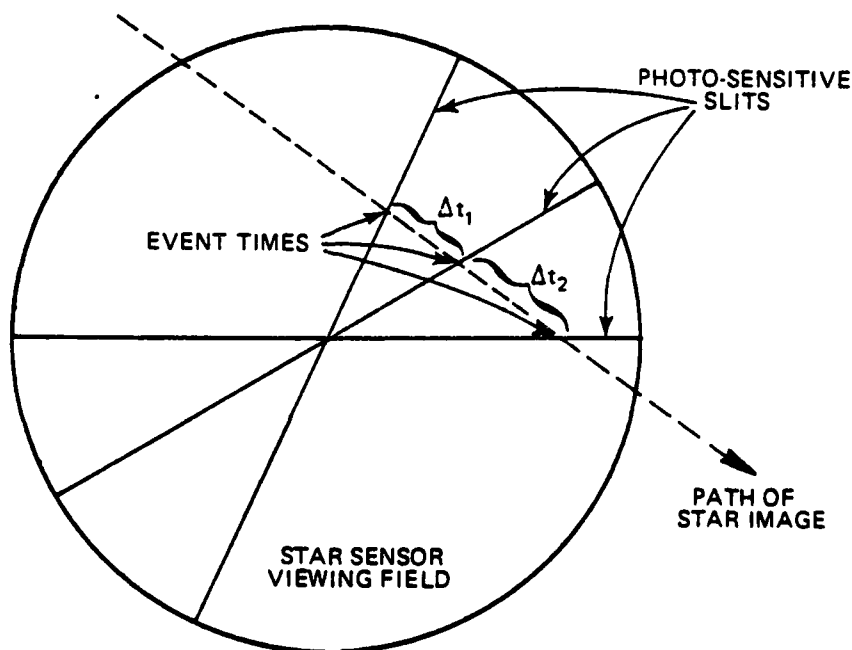


Figure 2.3-26 Spinning Vehicle Star Sensor

Correlation Guidance - Correlation guidance, which includes various terminal homing guidance techniques discussed in the next section, is a navigational process based on recognition of terrain features. These correlation techniques compare a sensed profile of ground signatures\* (of both man-made and natural objects) acquired during flight with signatures obtained from a reference map (whose position is known) prepared prior to flight. This comparison gives the relative position of the measured signature within the reference map, which is then used to create a position update for the inertial navigation system (INS) aboard the vehicle.

As a simple illustrative example of correlation guidance, consider a pilot in an aircraft who establishes his coordinates by noting a distinctive cluster of oil tanks below, which is also indicated on his navigator's chart. In effect, the pilot performs a pattern recognition which may be viewed as a comparison process.

In a more refined form, the same fundamental technique is used in image correlation schemes of map matching. With these schemes, the terrain is observed with imaging sensor (e.g., camera, laser scanner, radar imager) and compared with prestored reference maps until some best correlation of images or scene signatures is achieved between the observation and the reference. In either of these examples, a navigation fix can be achieved through the use of exhaustive search or scanning until the comparison between observation and reference indicates "best agreement" between the sensor observation and the map.

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\*The term signature refers to easily distinguishable or unique features in the imaged scene.

In the following development, a mathematical model of the correlation guidance navigational process is described. The model is oriented toward the TERrain CONtour Matching (TERCOM) correlation technique. In the TERCOM example, the altitude measurements are considered to lie on a straight line and are used to establish a particular terrain profile from which the vehicle's trajectory can be deduced through correlation.

TERCOM Technique<sup>(†)</sup> - The fundamental process employed in the TERCOM profile correlation technique involves a comparison of the profile of terrain heights overflown by the vehicle with a stored two-dimensional map of terrain heights. Through proper interpretation of such comparisons, over some profile of terrain, a measure of navigation system error is obtained. More specifically, the technique is stated as follows (Fig. 2.3-27):

Given:

- A profile of  $n$  radar altitude measurements  $A_{\text{Rad}}(j) = 1, 2, \dots, n$ , of vehicle altitude above some "radar surface" of the terrain
- Inertial navigator indications of vehicle position at times corresponding to the altitude measurements above
- A stored map (ADX Map; see below) of the theoretical radar surface elevation of the terrain which includes, as a subset, the actual vehicle path.

Determine:

- The vehicle position (navigation fix) which is most consistent with the above-described information regarding the vehicle path as well as the current set of profile measurements,  $A_{\text{Rad}}(j)$ ,  $j=1, 2, \dots, n$ .

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(†)This section contains material at a more advanced level than the rest of the text.

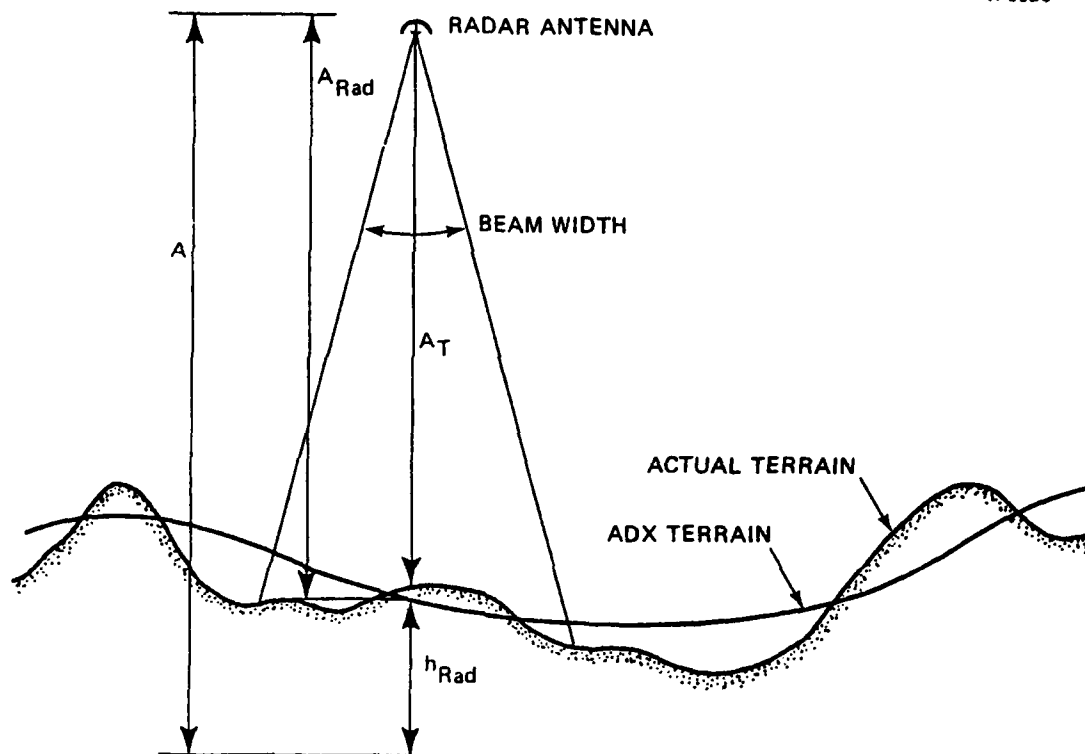


Figure 2.3-27 Radar ADX/Terrain Geometry

The radar surface is the apparent surface which the radar senses and is generally distinct from the actual terrain surface due to the effect of a wide beam radar antenna -- see Fig. 2.3-27. The theoretical radar surface is computed from a detailed simulation of the radar return pulse, and is generally denoted as the Area Depth Transformation (ADX) surface. One complication encountered in TERCOM reference map preparation deserves note. Methods for obtaining the raw mapping data use stereoscopic imagery and topographic mapping systems. Terrain altitudes for the TERCOM references must be derived through the use of stereo compilation techniques.

As shown in Fig. 2.3-27, the Radar Terrain Sensor (RTS) measurements are actually measurements of altitude above the radar surface  $A_{Rad}$  as distinguished from the vehicle altitude above a standard reference (sea level), denoted by  $A$ , and

the vehicle's true instantaneous height above the terrain, denoted by  $A_T$ . The RTS measurement is equal to the radar altitude,  $A_{Rad}$ , corrupted by an unpredictable random measurement error (noise). This measurement error includes radar fading, radar receiver noise, etc.

Simultaneously, as samples of measured radar vehicle altitude are acquired, samples of indicated vehicle position are available from an inertial navigation system (INS). They are designated  $x_{INS}$ ,  $y_{INS}$ , and  $A_{INS}$ . These samples (which of course contain the INS errors) are stored as an array along with similarly stored samples of RTS measured altitude. The RTS measurements may now be subtracted from the INS-indicated altitude to obtain a profile of measured radar surface elevations, denoted  $h_{Rad}$ .

The radar surface elevations are sampled at distance intervals which match the resolution of the stored reference data. The radar observations are made over a specified profile length, typically on the order of 7 to 9 km. Flat areas typically require a longer profile length than rugged terrain, in order to insure that sufficient information is obtained to provide reliable correlation.

The profile of these measured radar surface elevations is correlated with all of the terrain profiles ( $h_{REF}$ ) contained in the appropriate stored reference map. The profile in the reference map which has the highest correlation with the measured profile is then assumed to be the profile actually measured, and the location of the first point of the correlated profile within the reference map is the estimated position of the vehicle, at the time when the corresponding altitude measurement was taken.

There are several algorithms for computing an index of correlation, which is, in effect, the measure of a profile fit. In each case, the index is generated for all combinations of offsets between the reference map profiles and the sensed profile data. One algorithm commonly used is the Mean Absolute Difference (MAD), defined as

$$(MAD)_{n,l} = \sum_{i=1}^M \left| (h_{Rad}(l+i) - \frac{1}{M} \sum_{j=1}^M h_{Rad}(l+j)) - h_{Ref}(n,i) \right| \quad (2.3-37)$$

where

M = number of samples in each of the profiles stored in the terrain reference map (a two-dimensional array M×N)

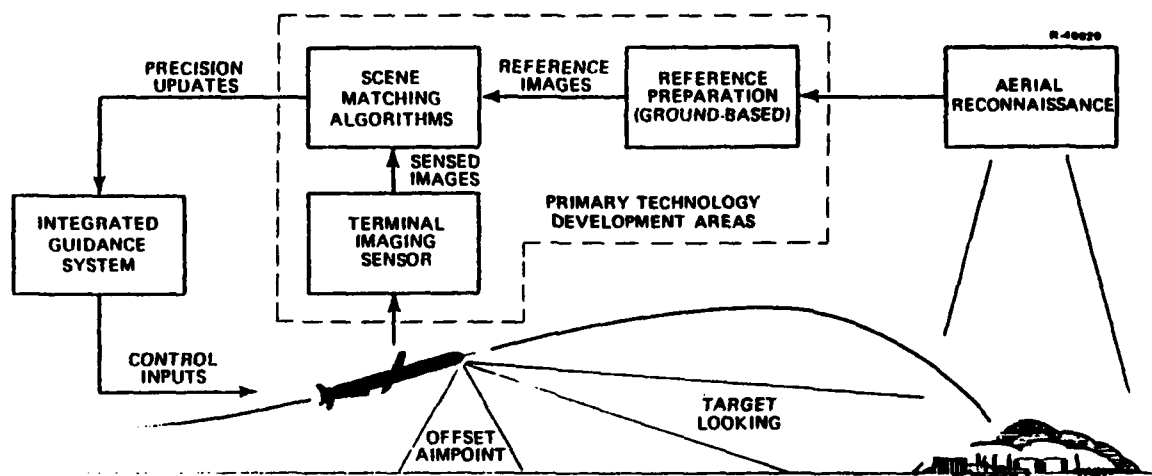
N = the number of profiles stored in the terrain reference map

l refers to a sample in the measured radar surface elevation profile (a one-dimensional array of total length L+M-1)

The MAD algorithm differences the means of the various parameters. Hence, the first parenthesis pair within the absolute value sign is the zero-mean version of the profile differences;  $h_{Ref}(n,i)$  has already had the mean removed. The result of this processing is a two-dimensional array, N×L in size, of values of the correlation index. The coordinates n,l corresponding to the minimum value determine the TERCOM navigational fix -- the best match between a portion of the measured terrain profile and various reference profiles.

### Terminal Homing

As can be seen from Fig. 2.3-28, there are three basic elements in the terminal homing process. The first is Reference



#### SYSTEM CONTEXT

■ FIXED TARGETS  
■ DUAL MODE

■ ADVERSE WEATHER  
■ DAY/NIGHT  
■ SYNTHETIC REFERENCES

Figure 2.3-28 Overview of Terminal Homing

Preparation (including pre-mission aerial reconnaissance). The second is Scene Imaging during flight, by the vehicle's onboard sensor. The third is Scene Matching or Correlation, in which the navigational position of the vehicle is determined. In the following discussion, Reference Preparation and in-flight Scene Imaging will be described together under Scene Generation. The Scene Matching process and algorithms that provide the precision updates to the vehicle guidance system are treated under Scene Correlation.

Scene Generation - Correlation guidance necessitates the compilation, and storage on board the vehicle, of a reference map of the imaged area. The onboard sensors generate an image that the correlation processor compares with the pre-stored reference scene. Navigation adjustments are based on how well the real-time imagery correlates in position with the predicted reference map.

As will be discussed in Unit Three (Source Data Collection and Remote Sensing) a variety of imaging sensors are available, covering many parts of the electromagnetic spectrum. However, many of these are specifically designed for weapon delivery systems (such as cruise missiles) operating at low altitudes and/or high vehicle speeds. Such missions often must take place in adverse weather. Because of the covert and/or classified nature of these missions, it is often not possible to obtain raw mapping data at the altitudes, velocities, and sensor wavelengths to be flown during the mission. The generation of these required reference maps is described below.

The most common form of raw source data for mission-dependent reference maps is stereoscopic optical photography. The characteristics of the various natural and man-made objects photographed in each scene must be determined. Very basically, these characteristics are the three-dimensional structure of the scene, and the nature of the materials comprising each element, feature, or object in the scene. Weather-related, seasonal, and diurnal variability must also be taken into account, as well as the possibility of camouflage. It is obviously desirable to have reference imagery that is as independent of the environment as possible.

Before the reference scene is regenerated for a particular wavelength and sensor type, its appearance at the viewing angle used by the operational sensor must be determined. This can be accomplished by projection of the original imagery along with knowledge of the scene's three-dimensional characteristics. Once the desired viewing angle has been obtained and the material characteristics of the components of a reference image scene have been determined, the scene can be regenerated as it would appear if imaged by a different sensor operating at a different wavelength. For example, a scene composed of a horizontal, black sheet of metal surrounded by



black asphalt, both at the same temperature, would look different in a passive infrared sensor image from the image provided by millimeter-wave radar. In infrared image, the more highly emissive asphalt would appear "hotter" (give off more radiation) than the metal. However, in the millimeter-wave radar image, the metal would give a much more intense return than the rougher and less-directionally reflective asphalt (only if the surface plane of the metal were nearly perpendicular to the incident radar beam). A more oblique angle might cause the highly directional signal return from the metal to appear much weaker than the unchanged, but less-directional return from the asphalt.

Note that adverse weather effects (such as haze, smoke, or clouds) between the ground and the collecting photographic system would degrade the resulting imagery. This, in turn, would decrease the likelihood of correctly characterizing the various surface materials in the scene. Change in material properties (such as reflectivity) as a function of viewing angle would also have to be taken into account before the final reference image is obtained.

The initial steps in this process -- material identification and scene regeneration at the new wavelength -- are usually carried out manually by a trained photographic interpreter, with the reprojection done by automated image-processing hardware/software. However, there are efforts underway to automate the entire procedure. For example, visible-light imagery taken at high altitudes and low nadir angles\* would be converted

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\*Nadir angle refers to an angle measured from a line passing directly downward through the earth's center. Low nadir angles, therefore, imply lines-of-sight oriented nearly vertically downward. Depression angle (a term used for atmospheric vehicles only) refers to an angle measured downward from the horizontal.

$$\text{Nadar Angle} = 90^\circ - \text{Depression Angle.}$$

automatically into the equivalent millimeter-wave radar image as seen from low altitudes and small depression angles.

Scene Correlation - Terminal-homing guidance systems based on scene-matching concepts use a sensor to gather imagery of scenes overflown enroute to a target or of the scene surrounding the target. This sensed information is processed in combination with prestored reference data to determine the location of the vehicle or its position relative to the target. Vehicle position relative to the target is determined directly for sensors that look at the target, or for those with offset aimpoints. The resulting estimated position relative to the target is integrated with other navigation data (e.g., from the inertial navigation system) to generate steering commands for the vehicle.

The general problem of matching a sensed image (gathered in real time) with prestored reference data (summarizing essential, predictable elements<sup>\*</sup> of the scene being sensed) is receiving greater and greater attention as the advantages of autonomous scene matching guidance become more evident to military planners. Unfortunately, the scene correlation problem is seriously complicated by the fact that the sensed imagery from many sensors (e.g., radar imagers, passive IR sensors, laser scanners) may be difficult to predict. The wavelength conversion problem was mentioned earlier. Another example is the difficulty in predicting imagery at the same wavelength. For instance, consider imagery of a building complex obtained from a high-resolution thermal scanner. Such imagery is dependent not only on the exterior dimensions of the building complex and its constituent materials (which may be assumed to be known during reference map preparation), but also on such

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\*Terrain height, radar reflectivity, etc.

unknown factors as internal building structure, heating/air conditioning of the buildings during the period prior to imaging, recent solar illumination history, cloud cover, surface conditions, fog/haze/smoke densities, etc.

While a direct prediction of the sensed imagery is usually not possible, there often are signature elements that can be predicted with high confidence and relied upon for accurate scene correlation. One of the most important problems to be addressed in the selection of a scene-correlation algorithm is the identification of those signature elements that can be predicted reliably for a given sensor/ imaging scenario.

Table 2.3-2 summarizes several levels of signature predictability ranked in order of quality. The table relates these levels to the appropriate scene correlation methods. For a given level of signature predictability, any algorithm appropriate to that level or lower could be used. Scene correlation approaches like the Minimum Integrated Squared Difference (MISD), Minimum Integrated Absolute Difference (MIAD), and Amplitude Ranking Techniques compare each raw pixel (picture resolution element) in the sensed image with the corresponding pixel in the stored reference image. Higher level scene correlation approaches involve some preprocessing of the overall sensed image (for example, to extract scene boundaries).

Although matching algorithms which assume a lower level of signature predictability (e.g., level 4 in the table) tend to be more sophisticated (and less easily deceived for their assumed level of signature predictability) they may, in some cases, provide poorer performance than algorithms recommended for higher levels (e.g., levels 1, 2, or 3 in the table) when used in a scenario for which a higher level of predictability is available. This can occur because the more sophisticated

TABLE 2.3-2  
LEVELS OF SIGNATURE PREDICTABILITY AND APPLICABLE  
SCENE-CORRELATION APPROACHES

LEVEL OF SIGNATURE PREDICTABILITY	APPLICABLE SCENE- CORRELATION APPROACHES
1. Image intensities can be predicted confidently with respect to an absolute standard	Minimum Integrated Squared Difference (MISD) Minimum Integrated Absolute Difference (MIAD)
2. Intensity differences can be predicted relative to an absolute scale but an unpredictable bias may exist	MISD with means removed MIAD with means removed
3. Relative intensities can be estimated (i.e., pixel i brighter than pixel j) but scale information is difficult to predict	Normalized Correlation Coefficient Algorithm Amplitude Ranking Techniques
4. Areas of different intensities can be separated but relative intensities are unpredictable (e.g., contrast reversals are common) and unpredicted signature elements may be present, including perspective distortions	Algorithms that preprocess the sensed images to extract predictable scene boundaries followed by correlation or feature-matching techniques which are insensitive to mispredicted feature elements or distortions

algorithms are designed to be insensitive to those signature elements that are assumed to be unpredictable. Thus, when used at a higher level, the algorithm does not take advantage of all usable information for determining the correct match location.

In addition to the problem of predicting sensed image intensities, scene-correlation algorithms must also be insensitive to image distortion arising from perspective variations.

This problem is particularly troublesome for scene-correlating scenarios having a requirement for low-depression-angle imaging of three-dimensional scenes (e.g., building complexes). For such scenes, unknown (or uncompensated) perspective errors can lead to a significant alteration of the sensed scene signature. A line-matching technique (requiring only level 4 signature predictability; see Table 2.3-2) is useful in this class of scenarios, and is described in some detail below.

Among the classes of correlation concepts that have been proposed for correlation guidance are

- Product algorithms
- Difference algorithms
- Recursive processing techniques
- Edge/feature identification techniques.

These concepts do not necessarily stand alone but can be imbedded in a processing chain. If used alone, product and difference algorithms attempt to correlate using all of the detail in the sensed data and, correspondingly, require reference maps that predict the scene as the sensor will see it. Recursive processing techniques estimate vehicle state parameters (e.g., position and velocity errors) by processing residual data generated by differencing estimated sensor data and actual sensor outputs. Edge detection techniques (including line-matching) strive to identify a unique feature in the sensed map.

Figure 2.3-29 illustrates the flow of information common to a wide variety of position updating systems that use correlation algorithms to compare a sensed signature (related to properties of the scene/terrain in the vicinity of the vehicle's ground path) and a stored reference map. The output

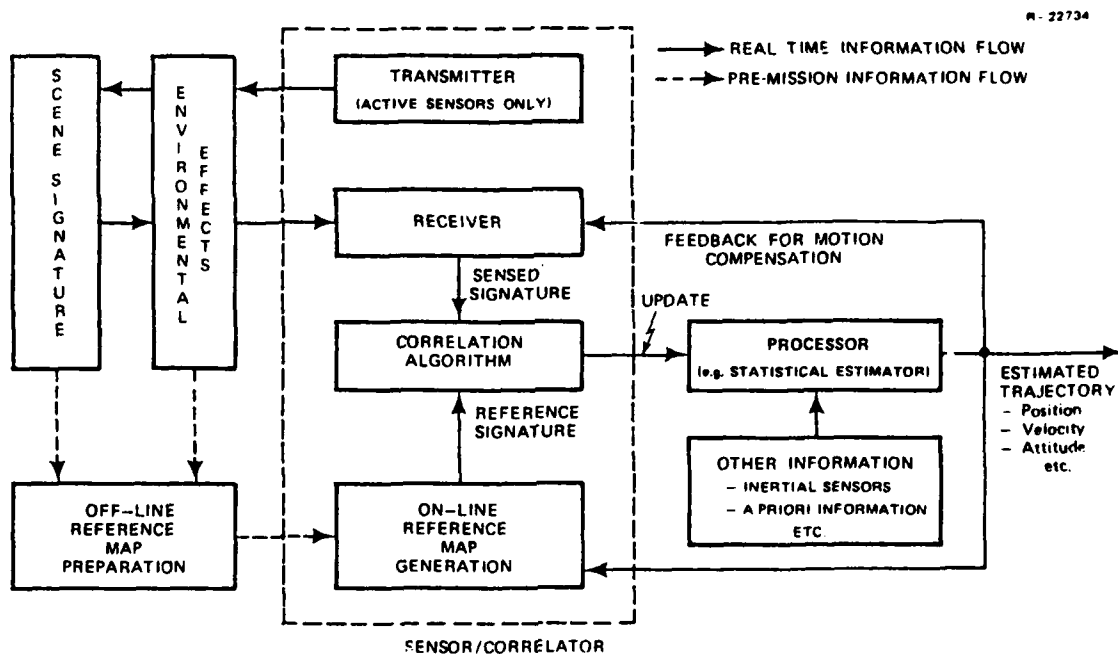


Figure 2.3-29 Generic Sensor Information Flow Diagram for Correlation Guidance System

of the correlation algorithm is typically combined with other available information (e.g., inertial sensors) using modern data processing techniques to produce an estimate of the state (position, velocity, attitude, etc.) of the vehicle.

Line matching techniques can be used to compare a set of stored reference lines (three-dimensional model) with a set of lines (two-dimensional model) extracted from a sensed image, in a manner that is insensitive to perspective. For illustrative purposes below, it is assumed that the lines from the sensed image have already been extracted. Thus, the only remaining problem is to match these as accurately as possible with the reference lines, while avoiding any unnecessary sensitivity to image distortions arising from scene relief.

Figure 2.3-30 illustrates a possible line-matching procedure, composed of both a coarse acquisition search and a fine search. Figure 2.3-31 illustrates the reference data required for this line-correlation technique. All lines which are expected to be visible from potential approach angles and present in the sensed signature are modeled. Figure 2.3-32 summarizes the edge information which would be extracted from the sensed image. The coordinates of one end-point plus orientation and length parameters completely define the extracted edges (also referred to as lines). The correlation process also uses several other parameters associated with each segment. The parameter describing the root mean square (RMS) uncertainty of the line's position in a direction orthogonal to its length is denoted by  $\sigma_{\rho i}$ . The RMS orientation uncertainty is given by  $\sigma_{\theta i}$ .

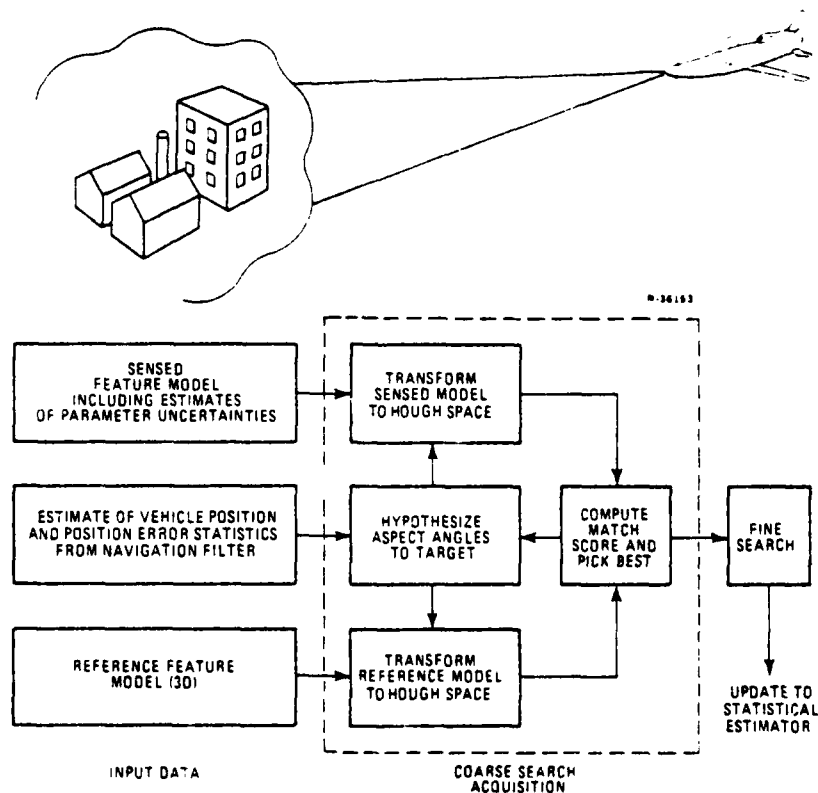
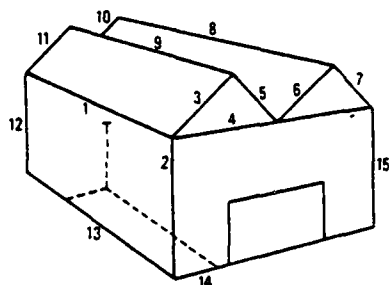


Figure 2.3-30 Example of a Line-Matching Procedure



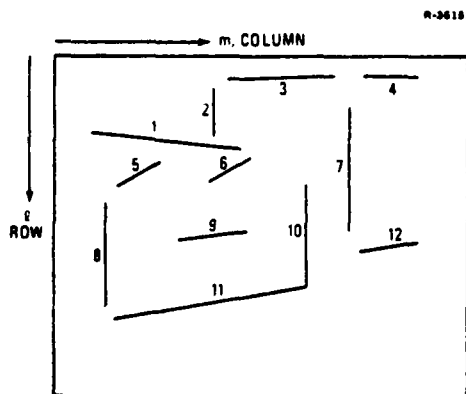
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(a) THREE-DIMENSIONAL WIRE FRAME MODEL OF SCENE SHOWING TARGET LOCATION (TYPICALLY INTERIOR POINT OF IMAGED STRUCTURE)

LINE NUMBER (j)	COORDINATES OF END POINTS OF MODELED LINES					
	$x_{j1}$	$y_{j1}$	$z_{j1}$	$x_{j2}$	$y_{j2}$	$z_{j2}$
1	—	—	—	—	—	—
2	—	—	—	—	—	—
3	—	—	—	—	—	—
⋮	⋮	⋮	⋮	⋮	⋮	⋮
J	—	—	—	—	—	—
TARGET POINT COORDINATES: $x_T, y_T, z_T$						

(b) TABLE OF REQUIRED REFERENCE DATA TO BE STORED ON BOARD MISSILE

Figure 2.3-31 Overview of Required Reference Data



(a) TYPICAL SENSED IMAGE AFTER EXTRACTION OF LINE FEATURES

LINE NUMBER $i$	END POINT COORDINATES		ORIENTATION AND LENGTH		ERROR PARAMETERS	
	$e_i$	$m_i$	$\theta_i$	$L_i$	$\sigma_{\theta i}$	$\sigma_{\rho i}$
1	—	—	—	—	SEE TEXT	
2	—	—	—	—		
3	—	—	—	—		
⋮	⋮	⋮	⋮	⋮		
I	—	—	—	—		

(b) TABLE OF SENSED FEATURE INFORMATION

Figure 2.3-32 Summary of Information (Hough Space Dimensions) Extracted from the Sensed Image



Hough space, referred to in Fig. 2.3-30, can be understood as follows. Once a set of aspect angles to the target (e.g., the doorway in the front of the building depicted in Fig. 2.3-31a) is hypothesized, the target coordinates in the sensed image can be determined from the known pointing angles of the sensor during imaging. With these hypothesized target coordinates as the transform origin, the sensed line features can be transformed to Hough space by calculating the distances,  $\rho_i$ , and the orientations,  $\theta_i$ , illustrated in Fig. 2.3-33. The associated uncertainties in  $\rho_i$  and  $\theta_i$ , denoted by  $\sigma_{\rho_i}$  and  $\sigma_{\theta_i}$  (e.g., see Fig. 2.3-32), are also calculated.

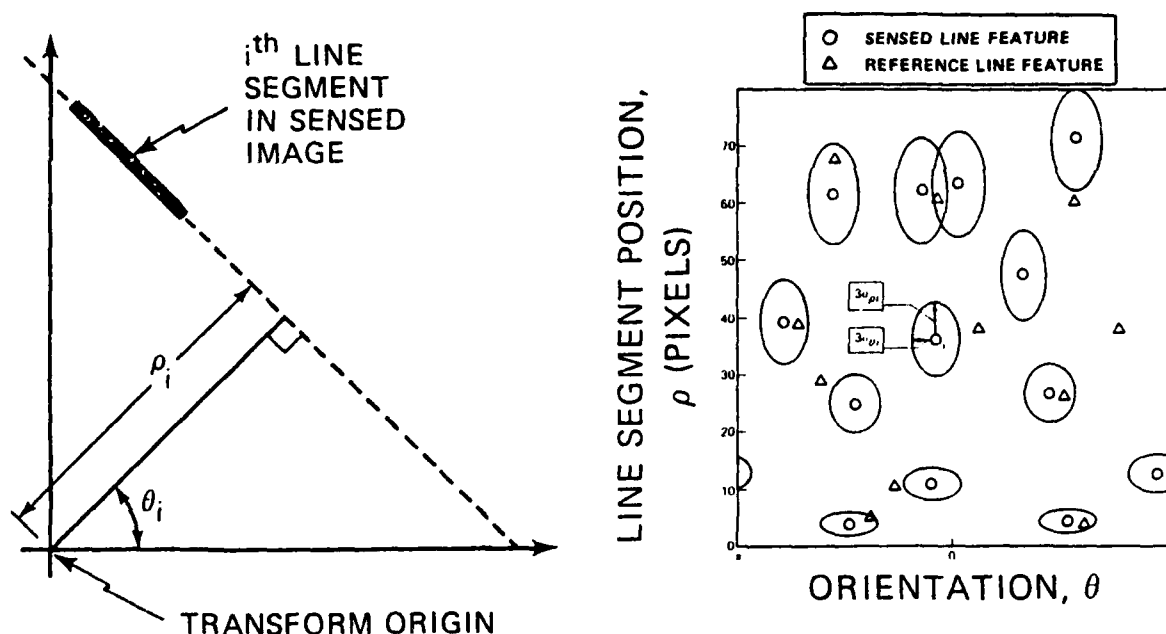


Figure 2.3-33 Illustration of Hough Transform for Straight Line Segments

Finally, it is assumed that the set of search parameters is specified by the navigation filter in the form of an estimate of vehicle position at the time the sensed image was collected. A set of associated position error statistics is

also generated. This information is used to define the boundaries of the search to be carried out by the coarse acquisition algorithm.

In summary, it should be stressed that there are a variety of correlation techniques and algorithms, as well as a variety of mission and imaging scenarios. Determining the best algorithm for a particular scenario or navigation requirement takes careful consideration of such factors as image resolution, scene signatures, and onboard digital storage and computational capability.

Radio Navigation - Radio navigation is a means by which the mobile user can exploit the constancy of the speed of light to derive his position relative to a known reference point. This can be accomplished actively by the user transmitting a coded signal and measuring the time required for the signal to be returned from a reference station. Position determination can also be done passively by measuring the time of arrival (TOA) and signal characteristics of transmissions broadcast at regular intervals from reference stations. Although it is convenient to think of these reference stations as being ground-fixed at precisely surveyed locations, they can be mobile (e.g., satellites) or even located at unknown positions. The important point is that radio navigation provides position fixes relative to the reference station position. The ability to translate this information to absolute (e.g., geodetic) coordinates is contingent upon knowledge of the position of the reference station at the time when the transmission was made.

There are a number of currently operational radio navigation systems and several more scheduled to become operational in the 1980s. Table 2.3-3 identifies the major systems

TABLE 2.3-3  
CHARACTERISTICS OF RADIO NAVIGATION AIDS

SYSTEM	FREQUENCY	FIX-TAKING TECHNIQUE	COVERAGE	RMS FIX ACCURACY
Nondirectional Beacons	200-1700 kHz	p-p	area	4-8 nm (2-D)**
VOR	108-118 kHz	$\alpha$ - $\alpha$	area	2-4 nm (2-D)**
TACAN	960-1215 MHz	p- $\alpha$	area	0.5-1 nm (2-D)**
VORTAC	108-118 MHz 960-1215 MHz	p- $\alpha$	area	1.0-2 nm (2-D)**
LORAN C	90-110 kHz	hyperbolic	land and coastal	100-500 m (2-D)
OMEGA	10-14 kHz	hyperbolic	global	2000-4000 m (2-D)
TRANSIT	150 & 400 MHz	Doppler	global	200 m (2-D)
GPS <sup>†</sup>	1575 & 1228 MHz	p-p <sup>*</sup>	global	10-20 m (4-D)*
JTIDS <sup>†</sup>	960-1215 MHz	p-p	portable	30-50 m (2-D)
PLRS <sup>†</sup>	420-450 MHz	p-p	portable	30-50 m (2-D)

\*Time and range-rate also provided

\*\*50-100 nm from beacons

†Systems currently under development.

and summarizes the operational characteristics of each. The operational characteristics of such systems can generally be described in terms of four basic parameters:

- Fix-taking techniques
- Propagation characteristics
- Coverage
- Accuracy.

A brief discussion of these parameters is provided below.

Radio navigation with respect to a specific reference station generally provides only a single position datum, either range ( $\rho$ ) or bearing ( $\alpha$ ).<sup>\*</sup> Thus, generation of a radio navigation

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\*Range-rate (velocity) information may also be provided.

fix generally requires measurements from multiple reference stations -- an exception being the Transit satellite system which provides a position fix based upon multiple measurements to a single, moving (satellite) reference station. Since most radio navigation systems\* provide only two dimensional fixes, the following discussion of fix-taking techniques is restricted to consideration of horizontal positions on the earth's surface.

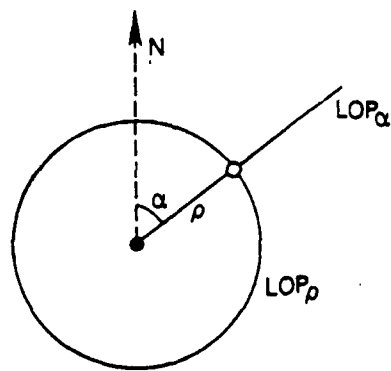
With knowledge of either  $\alpha$  or  $\rho$  to a reference station, lines of position (LOPs) can be plotted on a map. The intersection of two LOPs provides a navigation fix, although a third LOP is required for some systems to resolve potential ambiguities. The accuracy of the fix is a function of both the measurement accuracy and of the relative orientation of the LOPs. The accuracy of the fix is greatest if the LOPs intersect at right angles and is poorest when the LOPs are nearly parallel. Position fix accuracy is generally related to measurement accuracy by computing a quantity known as the Geometric Dilution of Precision (GDOP) for a given fix geometry:

$$\text{Position Error} = \text{GDOP} \times \text{Measurement Accuracy} \quad (2.3-38)$$

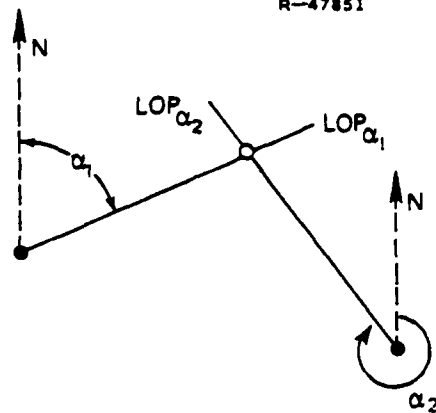
Some of the more common fix-taking techniques are shown in Fig. 2.3-34. The  $\rho$ - $\alpha$  approach employed by TACAN and VORTAC provides a unique position fix from a single station - the range measurement reduces user position uncertainty to a circle ( $\text{LOP}_\rho$ ) about the reference station and the bearing measurement provides a unique position ( $\text{LOP}_\alpha$ ) on that circle. Furthermore, the LOPs always intersect at right angles to yield optimum fix accuracy. Similarly, the  $\alpha$ - $\alpha$  technique achievable

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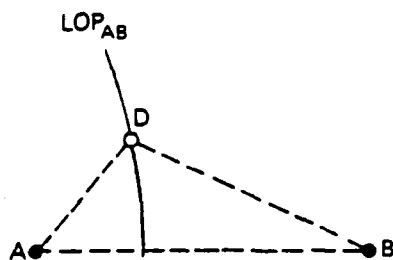
\*The Global Positioning System (GPS), a satellite network discussed in the next section and in more detail in Unit Four, is an exception.



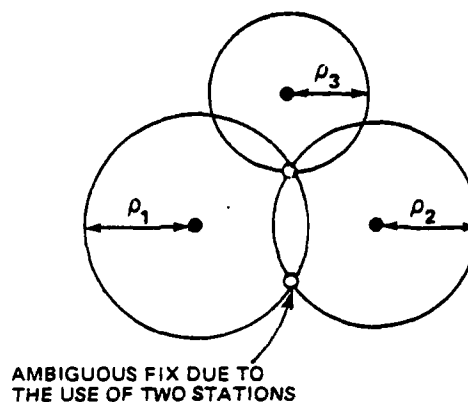
(a)



(b)



(c)



(d)

Figure 2.3-34 Common Fix-Taking Techniques: (a)  $\rho$ - $\alpha$ ; (b)  $\alpha$ - $\alpha$ ; (c) Hyperbolic; (d)  $\rho$ - $\rho$

with two VOR beacons yields a unique solution from fixes to two stations; however, the position fix accuracy is a direct function of the LOP geometry.

The Loran C and Omega systems are based upon measurements of the range difference<sup>\*</sup> between transmissions from two reference stations. With reference to Fig. 2.3-34 the LOPs are described by equations such as:

<sup>\*</sup>Omega measures phase difference rather than range difference, but the geometric considerations are the same. Reliance on phase difference techniques introduces position fix ambiguities that can be resolved only if the user knows his position at some initial time and then monitors Omega transmissions at regular intervals.

$$LOP_{AB} = k = Range_{BD} - Range_{AD} \quad (2.3-39)$$

The resulting LOPs are hyperbolas. Three pairs of stations (three LOPs) are generally needed for a unique position fix, although the possible ambiguity associated with two pairs of stations can often be resolved by the user.

The last fix-taking option of concern is the p-p technique employed by systems such as GPS, JTIDS, PLRS,\* and fixes from multiple nondirectional beacons. Generally, three measurements are required for a unique fix; however, the GPS geometry is such that ambiguous fixes are not an issue. More specifically, the ambiguous fix associated with GPS range measurements occurs in a region of outer space at a significant distance from the earth. In general, the best fix-taking accuracy (minimum GDOP) occurs for users within the triangle formed by the three reference stations and is reduced (greater GDOP) for users outside that triangle. Relative navigation nets such as PLRS and JTIDS can often provide this favorable geometry for some, but not all users; GPS users are always outside the triangle of optimum geometry because of the altitude of the satellites.

As previously mentioned, GPS is a four-dimensional system which provides altitude and time in addition to horizontal position fixes. Thus, four measurements are required for a unique GPS fix. The fourth (time) dimension is required for synchronization of user and GPS clocks. This synchronization requirement is typical of passive TOA systems like GPS.

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\*These acronyms represent: Global Positioning System, Joint Tactical Information Distribution System, and Position Location and Reporting System, all currently being developed under DoD sponsorship.

Implicit in the use of radio signals for navigation is the assumption that the radio path between the user and the reference station is direct and that the propagation velocity is known. Deviations from these assumptions can become the limiting factor in achievable fix-taking accuracy. Variation of the propagation velocity becomes an important consideration for the more accurate systems (GPS, LORAN) and corrections for velocity variations are often applied. Also of major concern are deviations from the desired propagation path. Such deviations arise in part from reflections off hills, buildings, etc. (multipath), but can also arise from interference between competing signal propagation mechanisms.

In free space, radio waves propagate in a straight line. Near the earth's surface, however, there are three possible propagation paths (see Fig. 2.3-35):

- Line-of-sight (LOS)
- Sky wave
- Ground wave.

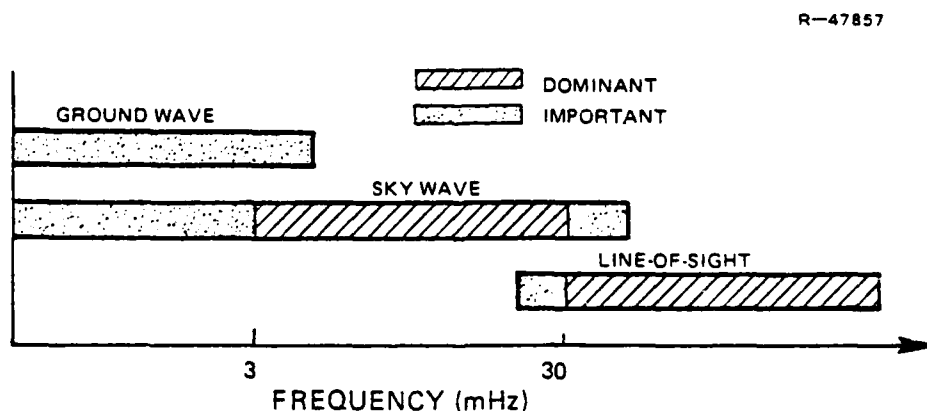


Figure 2.3-35 Relative Utility of Propagation Path Mechanisms for Radio Navigation

Skywaves are reflections of a radio signal from the earth's ionosphere. They are the dominant propagation mechanism for radio transmissions between 3 MHz and 30 MHz and an important mechanism for transmissions below 3 MHz. Note that multiple sky wave reflections (using the earth's surface as a lower reflector) can carry a signal over a major portion of the earth's surface. A major factor in the impact of sky wave propagation on radio navigation is the skip distance, which is the distance from a reference station to the point at which the primary ionospheric reflection first reaches earth. The skip distance, which is frequency-dependent, represents the minimum range from the reference station at which sky wave reflection becomes a possible propagation path. Although modeling and calibration data can be used to predict the direct propagation path for a sky wave transmission between a reference station and a user, propagation uncertainties typically limit measurement accuracy to no better than 2 km.

For transmission frequencies below 3 MHz, much of the transmitted radio energy follows the curvature of the earth. These ground waves tend to be the dominant propagation mechanism for low frequency radio transmissions out to a distance somewhat beyond the skip distance for the particular transmission frequency. Beyond the skip distance, the sky wave becomes dominant. Uncertainties in ground wave propagation characteristics tend to limit achievable ranging accuracy to about 100 m.

LOS is the dominant propagation mode for the radio navigation systems operating above 30 MHz (GPS, TACAN, VORTAC, JTIDS, PLRS). GPS satellites provide global coverage; however, LOS propagation limits the effective range of the other UHF systems to a few hundred km. LORAN C (100 kHz) uses ground wave propagation, with sky wave suppression equipment in the receiver, to achieve accurate measurements to a range of 1000



to 2500 km. Beyond that range, sky wave reception at reduced accuracy is possible. Omega relies almost exclusively on sky wave propagation.

In addition to the area of the earth's surface covered by a radio navigation system, the availability in time of the measurements is an important aspect of the system's usefulness. All of the systems mentioned in Table 2.3-3 except Transit can provide new measurements every few seconds for moving vehicles. Transit fixes are available only when a Transit satellite passes over a user, typically every one to four hours.

Application of Radio Navigation - Radio navigation systems can be used either as a stand-alone source of positioning information or as the source of external measurements to augment an inertial navigation system (INS). In either case, the potential user must address two fundamental issues:

- How to convert the measurements to a useful reference frame (datum)
- How to apply the measurements in an effective manner.

The proper solution to these issues clearly depends on the application. The options and considerations are illustrated in this section in the context of a specific example -- application of GPS to navigation updates for a cruise missile.

When fully operational, GPS will consist of a set of 24 satellites in 12-hour orbits at an altitude of about 20,000 km. At least 6 satellites will always be visible at any point on the surface of the earth. Each satellite can provide a range and range-rate measurement to the user (see Fig. 2.3-36).

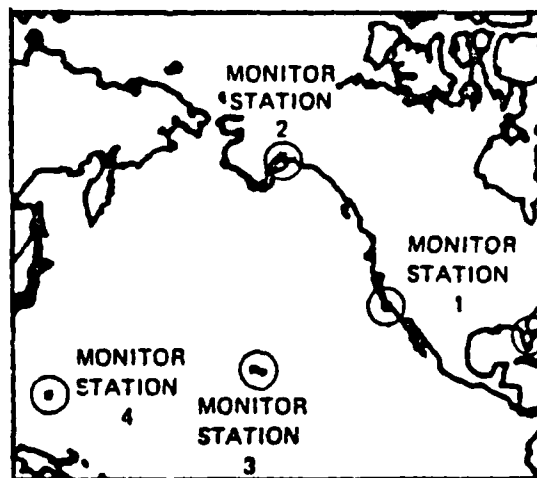
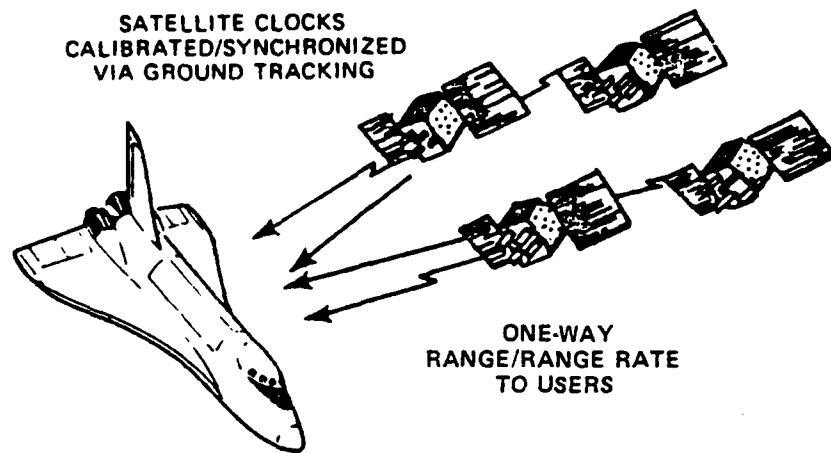


Figure 2.3-36 Elements of the NAVSTAR Global Positioning System (GPS)

Conversion of these measurements to useful navigation information requires knowledge of the position and velocity of the satellites in a convenient reference frame at the time when the measurements are taken. The necessary information is provided via the GPS Control Segment.

The GPS Control Segment consists of four Monitor Stations, each of which is precisely located in WGS-72 geodetic coordinates (Fig. 2.3-36). Each Monitor Station measures range and range-rate to all visible satellites at regular intervals. From these measurements, the Control Segment generates precise predictions of ephemerides\* and atomic frequency standard (clock) errors for each satellite. These predictions are reformatted into a special navigation message and uploaded to the appropriate satellite. The navigation message is then transmitted to the user by modulating the ranging signal with the data (encoded). Once decoded, the navigation message allows the user to compute satellite position (and velocity) in WGS-72 coordinates at the measurement time to a precision of a few meters. Subsequent translation to other geodetic datums can be accomplished by user software, as necessary. For a missile, the appropriate datum is clearly the one in which the target coordinates are provided.

The second issue to be addressed is that of effectively applying the radio navigation measurements. For a stand-alone radio navigation system, the problem is one of constructing a position fix using the  $\rho$ - $\rho$  techniques previously discussed. The computation may be complicated by such factors as vehicle acceleration, particularly if the four necessary GPS range measurements are taken sequentially rather than simultaneously. Dead-reckoning navigation filters which estimate vehicle acceleration based upon sequences of GPS measurements are one possible algorithm; Inertial Navigation Systems (INS) provide another option, as is further discussed below.

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\*An ephemeris (plural, ephemerides) is a tabulation of satellite position and velocity as a function of time.

Cruise Missile Example - GPS measurements can be used to update an INS. The INS provides a direct measurement of vehicle acceleration so that, in principle, a much greater position fix accuracy can be obtained than with the stand-alone mode. In addition to providing a position fix, however, GPS range-rate measurements are of sufficiently high quality to permit in-air alignment and calibration (see Section 2.3.5) of an INS characteristic of cruise missile applications. To achieve this in-air alignment and calibration capability, GPS measurements must be combined with INS outputs in an efficient manner using advanced filtering techniques.

The ability of GPS to improve the alignment of a cruise missile INS is illustrated in Fig. 2.3-37. In this example, the missile is equipped with an INS which possesses an accuracy of one nautical mile (1.852 km) per hour (gyro drift rates of 0.015 deg/hr) and an initial azimuth alignment error of 5 min. The navigation filter (refer to Section 2.3.5) is a 9-state statistical estimator which uses GPS updates of the INS to estimate:

- Position (3 states)
- Velocity (3 states)
- INS azimuth misalignment (1 state)
- INS level axis tilts (2 states).

As long as the missile flies a straight trajectory, the azimuth alignment error will not affect the INS output (see Section 2.3.2) and thus cannot be estimated by the navigation filter. In the example, the missile makes a right-angle turn during the launch phase and initiates two  $0.5 g^*$  maneuvers at 25 min.

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\*The  $g$  unit is equal to  $9.8 \text{ m/sec}^2$  (nominal acceleration of gravity at sea level).

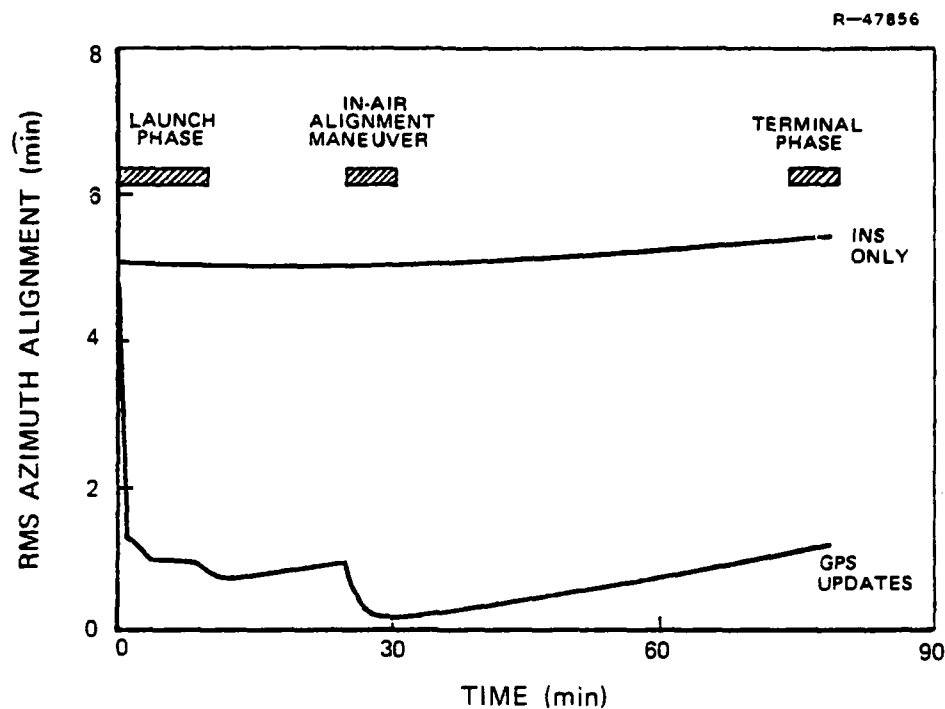


Figure 2.3-37 Example of In-Air Alignment of Cruise Missile INS with GPS

During each of these turning maneuvers, the azimuth alignment error introduces errors into the INS acceleration (change in velocity) output. By using GPS range and range-rate measurements, the navigation filter is able to eliminate the change in velocity errors from the navigation solution and, additionally, estimate the azimuth misalignment error itself. Following the maneuvers, the alignment error is reduced by a factor of 10. Note that between and following the maneuver sequences, the azimuth alignment error grows at a constant 0.015 deg/hr rate due to the gyro drifts. The importance of this in-air alignment is that in subsequent mission phases (during which it might not be desirable or possible to use GPS updates) the INS is capable of providing unaided inertial navigation system accuracy which is significantly better than if there had been no in-air alignment.

## CHAPTER FOUR

### DoD MAPPING AND CHARTING OPERATIONS

DoD mapping and charting activities cover a broad range of operations and produce a wide spectrum of products supporting weapon system operation and test applications. An overview of the kinds of products that are currently generated, the processes by which they are formulated, and the general direction of change in this critical area of DoD operations is presented in this section. The focus is on image-like product types, since gravity products have been treated previously and bathymetric products are not within the scope of this course.

As discussed in Unit One, U.S. military MC&G production is the responsibility of the Defense Mapping Agency (DMA). Using high-altitude aerial photography as the basic input material, supplemented when necessary by ground photography, intelligence reports, foreign maps, and other auxiliary information sources, DMA produces a variety of standard products in accordance with requirements established through the Office of the Under Secretary of Defense/Research and Engineering (OUSD/R&E) by the military users of these products. A few examples are of interest.

Standard topographic maps are produced by DMA in large quantities over most areas of the earth outside North America. Scales typically range from 1:250,000 (medium-scale, large-area maps) to 1:50,000 (large-scale maps - the military equivalent of United States Geological Survey (USGS) 15-minute quadrangle maps familiar to hikers in the U.S.). However, map scales outside this range are also encountered. Topographic maps are general-purpose cartographic products used in a wide variety of planning applications.

Special-purpose map sheets showing particular types of information used for specialized applications are also produced in large quantities. Typical of cartographic products in this category are the Joint Operations Graphics (JOGs), used in the planning and execution stages of military operations involving specific force components, and portraying data types of special interest to these force components.

Point Positioning Data Bases (PPDBs) are different from the cartographic products described above, although their applications in U.S. military operations are basically similar. PPDBs consist of stereo pairs of 9" x 9" high-altitude photographs (chips) that have been linked to an earth-based coordinate system, so that both the absolute and relative positions of points in the scene can be measured directly in the photographs. They are used primarily by forces in the field for tactical targeting purposes. In a typical application, two overlapping chips are set up in a piece of field equipment called the Analytical Photogrammetric Positioning System (APPS -- illustrated in Fig. 2.4-1). A target observe on a local reconnaissance photo is then located with respect to a landmark observable in the PPDB (road intersection, etc.). The target coordinates can then be determined.

Digital reference maps constitute a relatively recent product type that is likely to expand rapidly over the next fifteen years. These reference maps support applications of an advanced navigation concept known as correlation guidance (see Chapter 2.3) in emerging generations of strategic and tactical self-guided weapon systems. These reference maps are computer-compatible representations of geophysical features of the earth in specific locations along the intended path of the weapon system which provide a basis for inflight position updates. The information inherent in the features may be

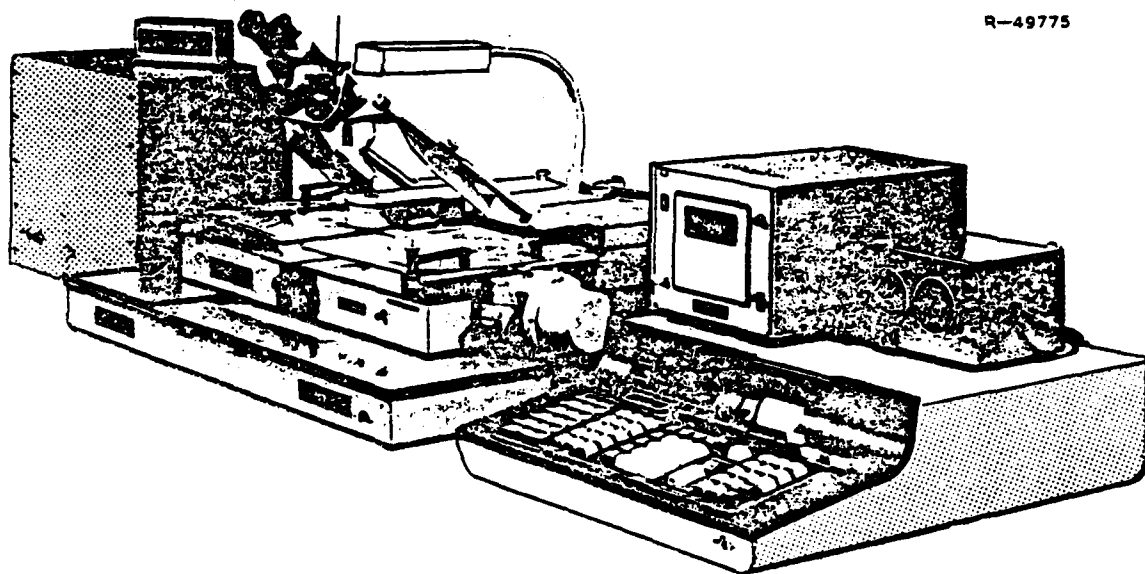


Figure 2.4-1 The Analytical Photogrammetric Positioning System

gathered in any of several domains (e.g., elevation profiles, radar reflectivities, microwave scatter, infrared emissions, optical scatter, gravity gradients) and in many alternative formats, depending on the needs of the weapon system's on-board sensor. Current active programs include compilation of terrain elevation matrices to support TERCOM\* applications (cruise missiles) and radar reflectivity scenes to support RADAG\*\* applications (Pershing II missile).

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\*TERrain COntour Matching.

\*\*RADar Aimpoint Guidance.



Another product type with strong similarity to the correlation guidance support products described above is the Digital Land Mass System (DLMS) data base. This is a large, ongoing compilation effort covering major portions of the North American, Asian, and European continents. It consists of a coarse grid of terrain elevation data, together with an identification and categorization of prominent geographic and cultural features that are expected to dominate the scope display of a low-altitude aircraft navigation radar. It is used to drive a mission simulator that presents the aircraft crew with a preflight view of what the scope returns will look like enroute to potential target areas. In this sense, it can be regarded as a near-term, manual, non-real-time application of correlation guidance principles to the manned bomber target penetration problem.

The foregoing examples are intended to illustrate representative types of standard mapping and charting products currently produced in large volume by the Defense Mapping Agency. There are many others that have not been specifically listed -- including strategic target coordinate sets, U.S. ICBM launcher position surveys, and many forms of information files. A complete compilation of the current DMA product list contains several hundred entries.

Prospective managers of advanced weapon system development programs which might require new types of MC&G products to support their operations should be aware that the lead times on new products are often very long, and their development costs are high. As an example, the TERCOM reference maps have been in preparation for several years and will not be completed until the late 1980's. The DLMS product set will require even longer.

The current mapping and charting production process within DMA is configured around the exploitation of photographic source materials. Principal inputs are in film form. Initial preprocessing of the film is aimed at establishing relationships between object coordinates in the photographs and object coordinates in a geodetic coordinate system. Terrain elevation data is then extracted from stereo pairs of photographs using highly automated parallax measurement techniques. This is followed by manual identification and extraction of the planimetric features, consolidation of the resulting data with other data already in the DMA files (previous versions of the map, etc.), and generation of the new or revised map. With this production process, the basic MC&G information files consist of the source materials and the finished maps. They are supplemented by product-specific files containing support or intermediate data associated with the respective products.

Key technological advances, however, together with evolving military needs for MC&G support, are changing the approaches to image data handling. Increasing use of digital technologies to replace film technologies, combined with the dominance of digital products provided by DMA, will lead to an increasingly digital MC&G production system over the next ten years. A highly probable view of DMA operations at the end of the next decade is shown in Fig. 2.4-2. In this situation, the central repositories of extracted cartographic and photogrammetric data are a set of unified and coordinated data bases that can be used to generate a wide range of general-purpose and special-purpose products ranging from conventional map sheets to highly specialized reports. Products, in this context, are viewed as particular assemblies of information from central data banks, packaged according to specific military needs. Textual, graphics, and digital products will be generated from the data banks by computer-controlled output devices under the

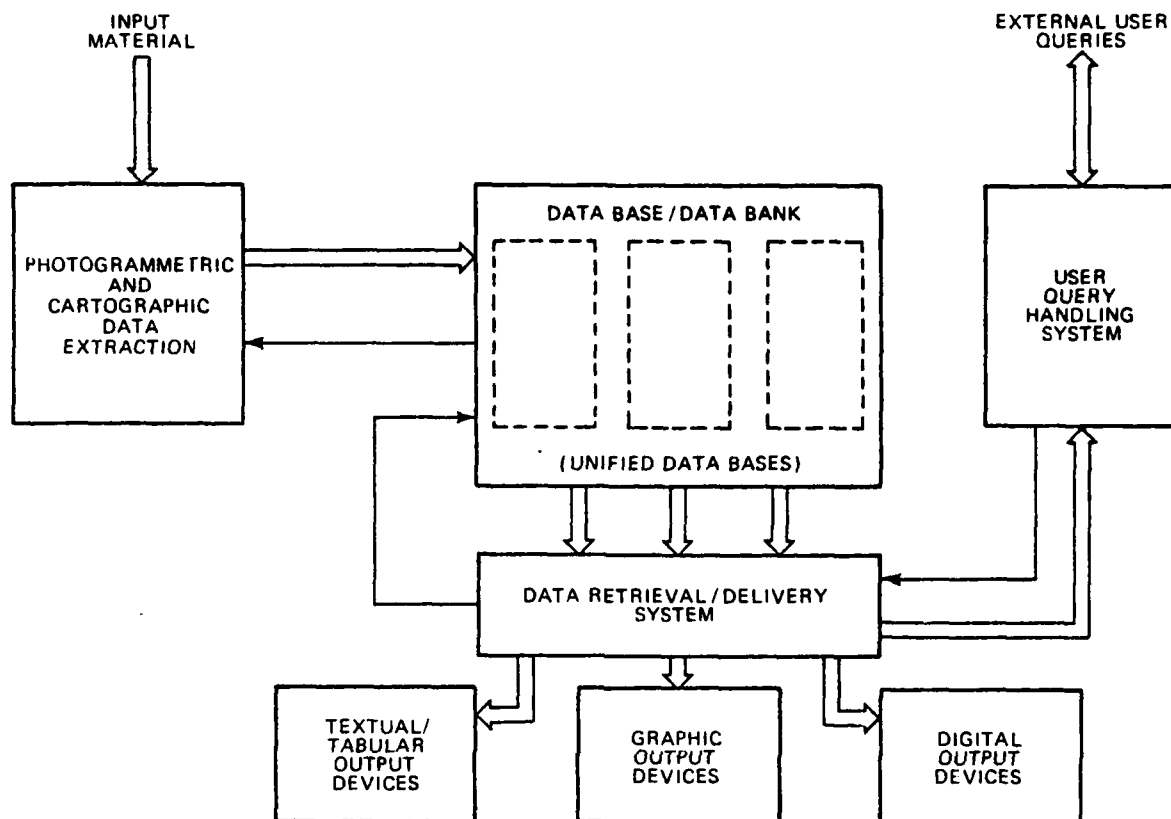


Figure 2.4-2 MC&G Data Storage and Delivery in 1990

control of a highly capable data base management system. Provisions will be made for handling ad hoc requests for access directly to the information base itself when standard products do not adequately support an inquiry.

From the viewpoint of future DoD users of MC&G data, the principal impact of this change may be summarized as follows:

- Much shorter lead times in responding to requirements for new or modified product types
- An ability to provide a wider variety of products tailored to specific applications, rather than emphasis on standardized products

- Provision for providing rapid "breadboard" versions of candidate new products to permit early user evaluation of alternatives before establishing firm production requirements
- Introduction of fast-response user query handling systems to permit direct access to the DMA data bank.

## CHAPTER FIVE

### DIGITAL DATA BASE CHARACTERISTICS AND EXPLOITATION

Modern digital computer technology has provided a wide variety of lightweight, low power digital circuitry and digital memory implementations which have direct application to airborne weapon systems guidance and control. In particular, the development of powerful single-chip microprocessors and associated special-purpose processing chips at low cost makes it feasible to provide extremely powerful computational facilities in expendable modules.

This dramatically improved processing and storage capability makes it possible to provide a weapon system with a machine-usable "map" of terrain, target features, and other significant information. Such a map is intended for direct use in conjunction with onboard sensors. The computing power supports complex algorithms and system combination concepts that make precise navigation, including alternate routing, a completely automatic process.

Figure 2.5-1 illustrates such a combination in terms of a general correlation guidance approach.\* The system uses onboard stored maps and locally-sensed data to provide position update information to the onboard inertial navigation system. The combination of inertial navigation (to provide precise steering over the preselected course) with highly accurate position updates (to bound the position error growth in the inertial navigator) results in fully automatic steering to the preselected target. As shown in Fig. 2.5-1, the weapon system can be expected to steer relatively complex courses between

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\*Section 2.3.7 provides a more detailed treatment of correlation guidance and related navigation system approaches.

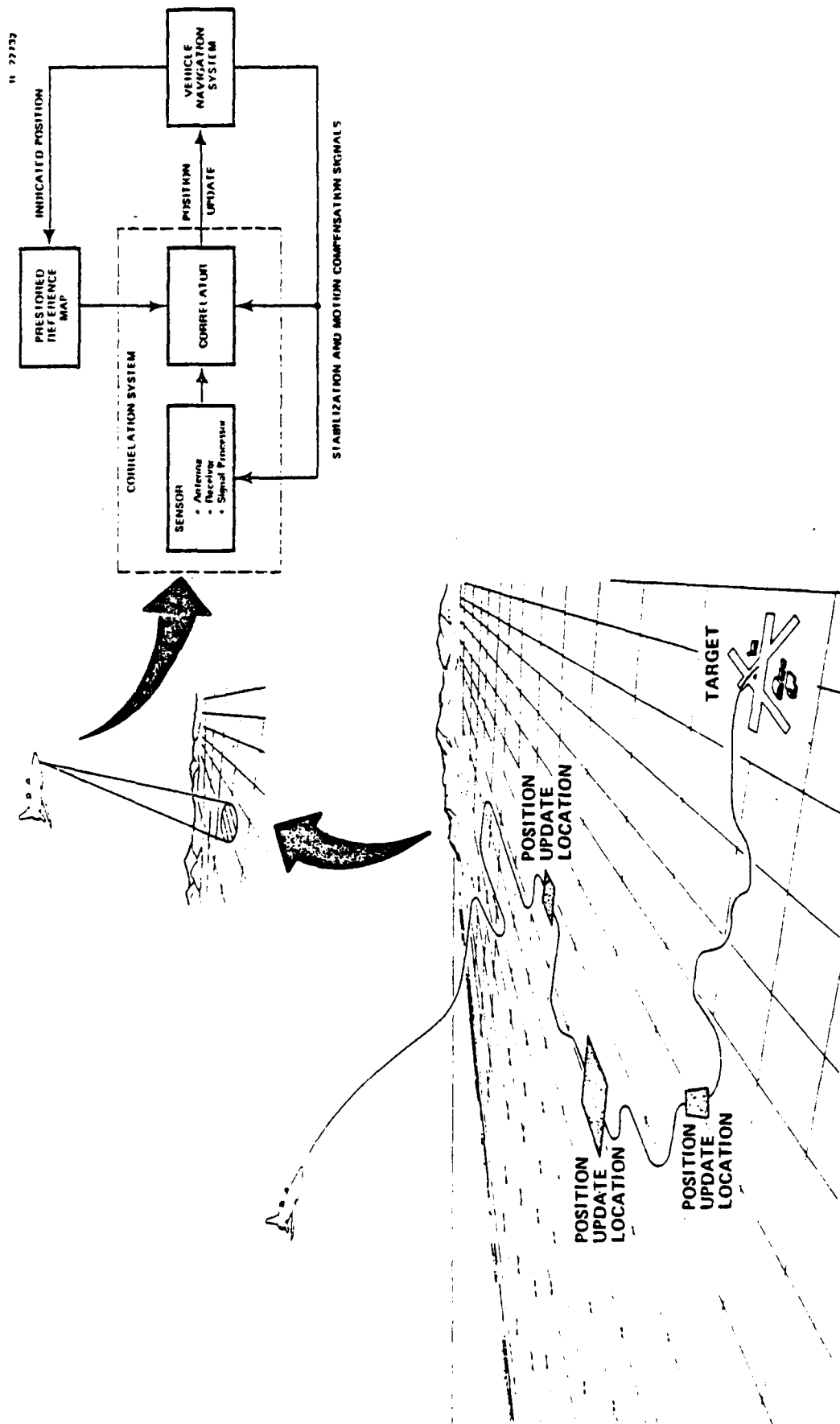


Figure 2.5-1 Generic Correlation Guidance System

position updates. Terrain-following maneuvers are used to avoid detection by ground radars. The relatively few position update locations can be carefully preselected for optimal scene-matching characteristics, depending on the particular correlation algorithms chosen, and therefore represent minimal memory requirements for the weapon system.

Similar concepts apply to terminal guidance for such devices as ballistic missile reentry vehicles. Figure 2.5-2 shows a similar application of the combined use of a locally-sensed map. In this case, the map is the terrain surface as it would be sensed by a radar altimeter. Again an inertial navigation system is used to provide precise local steering once the position update is accomplished. Note in this case that the ballistic trajectory of the system represents a fairly "clean history," i.e., the position is well known up to the terminal phase. Note, too, that local maneuvers are relatively simple and occur over a relatively short term, compared to the cruise missile system shown in Fig. 2.5-1. The constraints for terminal guidance are that highly accurate and very rapid local position updates are required. This implies the need for a fairly large-area, high-resolution initial position map, as opposed to a selection of relatively small area, optimized maps for the correlation guidance scheme.

Modern reconnaissance and high-speed digital analytical techniques provide the underlying capability to support the guidance concepts discussed above. These techniques have been under development for some years in the MC&G communities. A main thrust of digital data base generation has been to develop a worldwide, digital, standard elevation data base. This is an effort to support the mapping and charting processes and largely involves DMA. In addition, DMA is committed to provide a Digital Land Mass System (DLMS) capability, in which a highly

interactive man/machine process generates data bases for simulating radar scenes through analytical techniques based on optically-sensed imagery. These two activities have been the cornerstone of the capability to provide geodetically accurate representations of the earth's surface over a wide range of frequencies. In all cases, the resulting data bases (which are used to produce MC&G products) are digital representations of the earth's surface. The most common of these are:

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- Digital Terrain Models (DTM), in which high-degree polynomial representations of the local surface are generated, centered around the given datum points
- Digitized images, in which measured values of emitted or reflected radiation for a particular spectral band are stored for each grid element (pixel).

All of these data bases are directly usable for mapping and charting production operations. They constitute elements of the long-term development of unified digital data bases to support a highly automated production capability. In the context of weapon systems applications, however, these data bases can be used to support selection of specific data bases on board military vehicles. To accomplish this support it is necessary for mission planning personnel to have interactive access to the data bases. Backup computer processing is also required to assist them in the trajectory selection and scene selection update processes. The selected scenes, which represent subsets of the overall digital data bases, can be processed as appropriate to produce the local reference maps which are carried on a vehicle.

In some cases, these are simple subsets of the primary data bases. For example, if the resolution cells are small enough, a simple local elevation matrix may suffice for the terminal guidance concept illustrated earlier. In other cases, extensive computer processing may be required to convert the primary data into reference maps optimized for the sensors used by the given weapon system. The remainder of this chapter addresses the general considerations involved in developing digital data bases for weapon system applications. Detailed discussions of the precise mathematical processes involved are addressed in other chapters, as referred to in the following discussion.

### 2.5.1 Digital Data Base Development

Onboard digital data bases for weapon system navigation and guidance applications are generated from the same fundamental information as are conventional maps and navigation charts. For modern MC&G data, this information is contained in cartographic data bases containing three types of information:

- Fundamental source information (e.g., reconnaissance photographs, etc.)
- Extracted geodetic and feature data (e.g., terrain elevation matrices, feature-type/location collections)
- Geographic/cartographic information (e.g., political boundaries, place names, and other legends).

The class of fundamental source data includes conventional information such as gravity measurements, measurements of the earth's magnetic field, infrared imagery, and, at the current state-of-the-art, radar-frequency reflectivity tables. However, the bulk of the information required, both for onboard guidance and for mission-planning purposes, is contained in two types of data bases, photographic imagery and terrain elevation/surface material classifications. These are the two key classes for the actual operational use of MC&G data for weapon system guidance and control.

One subclass of cartographic data bases that is of extreme importance for successful mission planning and execution is the set of extracted data points or features known as control. These are collections of distinctive, accurately measured points on the earth's surface. Much of this location information has been obtained through direct ground measurement

over the years. However, modern surveillance and remote sensing techniques plus the greatly improved geodetic modeling of the earth's surface have made it possible to develop a very finely divided, heavily populated data base of control points. These control points are used in several ways in the production of digital data bases:

- Identification of specific control points in a reconnaissance photo or other raw source medium permits accurate geometric corrections and registration of the source to the earth's surface
- Identification of control points in the corrected source material permits accurate measurements to be made, relative to the control points, for other features of interest
- The control point network can be extended by including newly-observed candidate control points in the grid as a result of mensuration\* on the corrected and registered source materials.

The availability of corrected source materials, with control, for an area of interest to a particular mission then supports the extraction of waypoint, target, and position update locations for inclusion in onboard data bases. The techniques applied to extract the data are discussed in Units One and Three. The most important method of data extraction in support of weapon system guidance, control, and mission-planning is analytical triangulation based on stereo image pairs. The triangulation is used to develop tabulations of local terrain-height data.

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\*Mensuration is the process of making highly accurate measurements of the relative positions of objects on an image.

Research and development currently underway in a number of institutions suggests the possibility of replacing the highly-detailed elevation tabular approach with Digital Terrain Models. Figure 2.5-3 illustrates the general concepts behind these models. In the case of polynomial approximation, the theory provides for some form of curve fitting to the local earth surface at specific points of interest. For a reasonably wide range of territory the general form of the curve (that is, the number of terms in a polynomial expansion) remains the same, and the onboard data base need only contain the coefficients of the terms for the specific points. Expansion can be done onboard in real time, to develop a reference map.

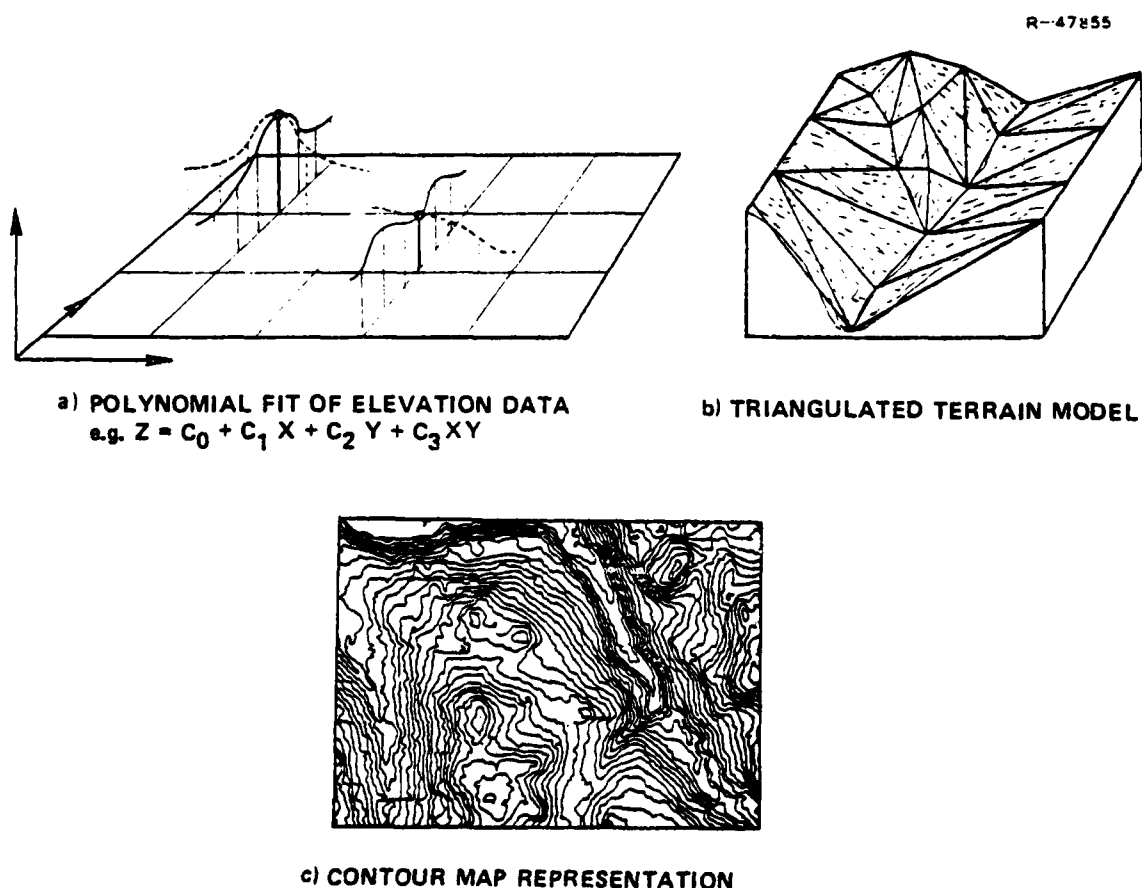


Figure 2.5-3 Alternative Digital Terrain Model Concept

The triangulation approach is based on the concept of approximating the earth's surface in a given area by a network of contiguous triangles. That is, the surface can be described by identifying as large a set of plane surfaces as is required. As shown in Fig. 2.5-3, this results in an irregular network of approximating figures, which poses additional computational burdens on the onboard system. However, current research indicates that the required total number of stored points (vertices of the defining triangles) is sufficiently small to allow reasonably small memories to contain the onboard maps.

Finally, the concept of storing contour lines equivalent to a topographic map is also a promising approach. In this case, the fundamental information is composed of chained series of points representing constant elevation contours in the local terrain. The underlying grid for application of the individual points is a regular rectangular coordinate system (e.g., UTM -- see Section 1.2.6 in Unit One). Measurements are then translated from the onboard sensors into their location in the onboard map.

For mission-planning activities, local terrain elevation data and the geographic/political boundary information must be available over all areas of interest to the mission. The reason for this is that mission planning requires the simultaneous consideration of many conflicting requirements. These are touched upon in Chapter Six, but a simple example will suffice to illustrate the point. Given that a particular target is to be approached by a cruise missile, consideration must be given to the following factors:

- Preservation of sufficient energy reserves in the missile to enable it to accomplish all required maneuvers and still reach the target

- Selection of an approach path which provides minimum probability of detection of the missile by ground forces at known locations
- Selection of a flight path which avoids all known obstacles and minimizes the probability that the weapon system (operating within its allowable guidance error) will crash into terrain or man-made objects.

The mission planner, therefore, must simultaneously perform several jobs. He must select an optimal-range trajectory as well as find a path which is hidden (in the line-of-sight sense) from defensive forces by the terrain. He must select an optimal altitude path satisfying the above two constraints and still minimize terrain-following energy consumption. This process involves simultaneous consideration of human-understandable imagery, local earth surface contours/features, and extensive computer support.

Translation of information from the source materials to the data bases may involve heavy computational loads and significant data processing support. This can occur for either the onboard application or the mission-planning activity. For example, development of a simulated radar view of a target area, based on the use of stereo reconnaissance photographs, demands extensive computation as well as the combination of auxiliary data bases with the imagery. Figure 2.5-4 indicates the complexity of such a process for a modern-day synthetic aperture radar. This type of airborne radar depends upon onboard computer processing of time-sequenced radar probes to form a two-dimensional image. The vehicle trajectory as determined from inertial navigation or other navigational inputs is a vital part of the computation. This information is combined with the antenna pattern data and the expected radar reflection characteristics of the earth's surface to produce a

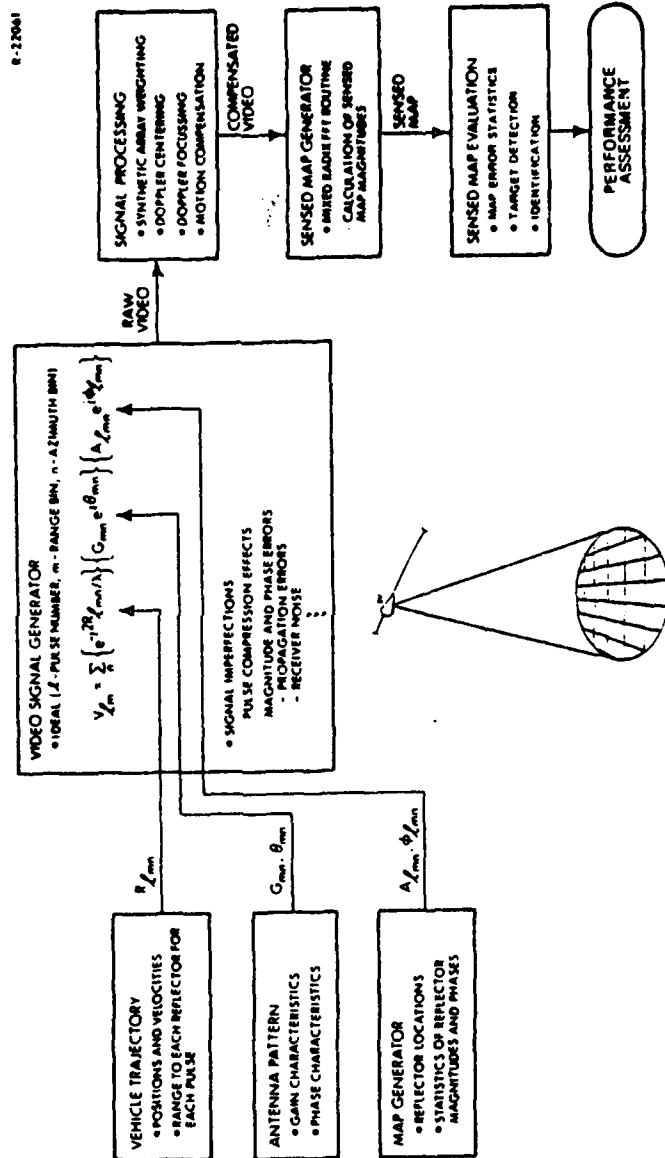


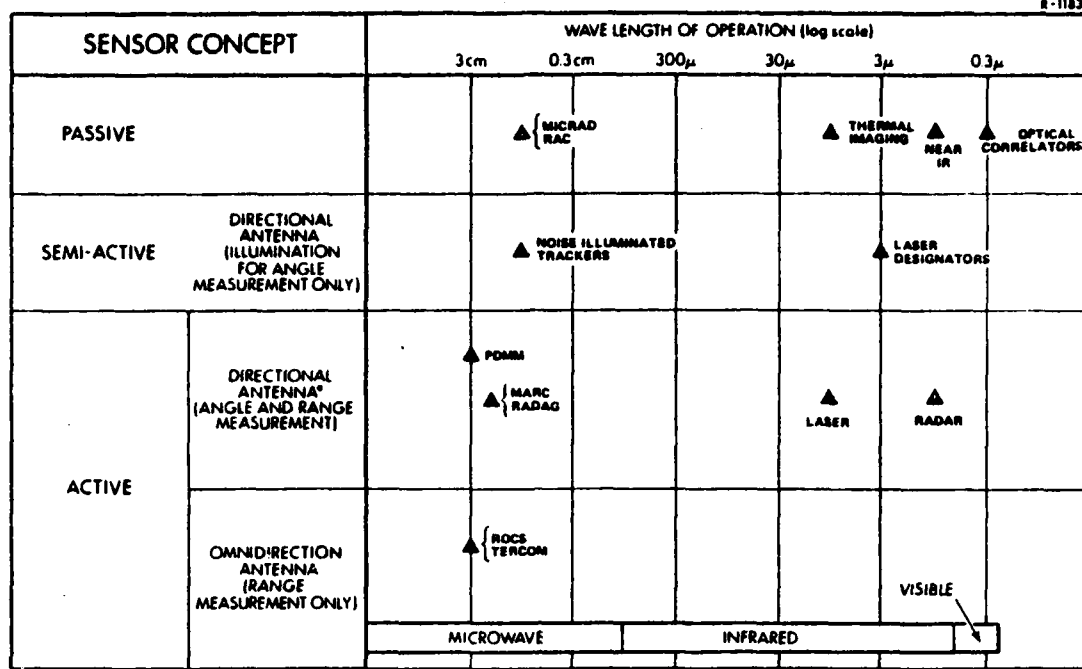
Figure 2.5-4 Synthetic Aperture Radar Simulation (SARSIM)

simulated video signal. A key part of this input set is the determination, typically by a human operator, of the surface material classifications and orientation of the principal radar reflectors in the scene. This information is not typically obtained from radar signals. Instead it is gathered by a trained, experienced photointerpreter utilizing classification rules to label the important areas. As currently conceived, the human operator's interpretation of the scene is provided to a digital computer. The format is digitized graphic overlays on the specific scenes. The output of the total process is a map for the target area which can be used either to produce real-time training displays for pilots or to serve as an onboard reference map for automatic guidance.

The computational workload and the degree of human involvement in the mission-planning stage varies from sensor to sensor. It also varies according to the fundamental sensed signals used in a given navigation concept. Figure 2.5-5 illustrates the possible range of sensor concepts that might be applied to the process of correlation-based navigation. Issues that must be addressed in utilizing a specific sensor include the frequency of operation of the sensor and whether or not it is active (i.e., radiating). (The other possibility is a purely passive remote sensing device.) In the case of an active or semi-active device, the risk of revealing the vehicle's presence is high. However, the accuracy of the sensed information tends to be greater. Such systems provide much better correlation with the sensed scene at the surface. They consequently provide a more accurate and more manageable flight trajectory.

Another major consideration is the frequency range of operation in the imaging sensor. The more closely this approaches the normal optical spectrum, the easier it is to derive information





\*REAL BEAM OR SYNTHETIC ARRAY

Figure 2.5-5 Correlation (Imaging) Based Homing Sensors

for the onboard map from optical forms of reconnaissance data. As indicated in Fig. 2.5-4 in conjunction with synthetic aperture radar simulation,\* the use of information recorded at one spectral range to predict the performance of a sensor operating in a different frequency range requires a great deal of analytic and computer support. The prediction also adds an unavoidable dimension of error to the onboard map. The additional ambiguity increases the difficulty of selecting a position update or target area map which can be correctly correlated with the actual surface scene.

\*Also see Terminal Homing and Scene Correlation in Section 2.3.7.

In summary, the specific correlation system concept, including the correlation algorithm and the fundamental sensor complement, determines the best form and requirements for onboard data bases. Most current systems utilize direct measurement of terrain height and can therefore be satisfied by an onboard terrain-height matrix. Extension of the concept of correlation guidance to other, more exotic sensors requires both significant data processing and analytical support to prepare the maps. Also, more powerful onboard computational capability is required to compensate for problems of new, unavoidably induced error mechanisms. Table 2.5-1 illustrates the range of scene-matching techniques\* currently in use or contemplated for weapon system and other navigational applications. The table also indicates the potential strengths, weaknesses, and tradeoffs required to choose among the various approaches. All of the techniques included in the table are based on utilization of imaging sensors. Other forms of geophysical signature-matching concepts are also under consideration. For example, the use of gravity and magnetic field measurements is being investigated as an additional element of the sensor complement. Table 2.5-2 illustrates some of the key considerations and prospects for such geophysical signature matching concepts.

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\*Refer to Section 2.3.7 for a discussion of the general scene-matching problem.

TABLE 2.5-1  
SCENE MATCHING TECHNIQUES

SCENE MATCHING TECHNIQUE	APPROACH TO IMAGE MATCHING	PRIMARY STRENGTHS	PRIMARY WEAKNESSES
Transform Coefficient Matching	Mean Absolute Difference (MAD) processing of reference and sensed image Hadamard coefficients	Enhances peak sharpness Reduced sensitivity to random noise Has been developed with geometry preprocessor	Requires gray level predictions Increased computational requirements
Address Modification Correlation	Images matched over their entire frame through the detection and correction of image warps	Exact full frame registration of planar scenes Simultaneous detection and correction of warp coefficients	Requires gray level predictions Sensitive to three-dimensional perspective distortions
Optical Matched Filter	Analog, matched-filter system using coherent light processing	Essentially instantaneous correlation Extremely large memory capacity Parallel processing of many reference images	Requires gray level predictions Processing techniques restricted by hardware
Three-Dimensional Surface Shell Correlator	Matches stored three-dimensional target model with data obtained from active ranging sensor	Requires no gray level predictions* May accommodate all approach azimuths Resistant to intentional signature modifications	Requires ranging sensor High computational requirements
Relative Information Vector (RIV) Feature Matching	Match based on maximizing number of corresponding RIVs	Requires no gray level predictions* Incorporates three-dimensional scene geometry Insensitive to contrast reversals Minimum memory requirements	Subpixel accuracies difficult to achieve Feature extraction sensitive to noise
Model Matching	Matches feature based models made up of extracted line segments and vertices	Requires no gray level predictions* Insensitive to contrast reversals Minimum memory requirements	Subpixel accuracies difficult to achieve Feature extraction sensitive to noise

\*RIV characterizes each feature based on arrangement of nearby features.

\*Implies significant reduction in reference preparation requirements.

TABLE 2.5-2  
OVERVIEW OF GEOPHYSICAL SIGNATURE MATCHING CONCEPTS

GEOPHYSICAL QUANTITY	SENSOR NEEDED	KEY CONSIDERATIONS AND RISKS	PROSPECTS
Gravity Anomalies	Gravimeter (Specialized Vertical Accelerometer)	<ul style="list-style-type: none"> <li>• Not all areas have suitable RMS gradients</li> <li>• Accurate velocity needed for Eötvös correction</li> <li>• "Signal" masked by kinematic vertical accelerations</li> </ul>	<ul style="list-style-type: none"> <li>• Fair to good for marine applications</li> <li>• Poor for airborne applications because of kinematic vertical accelerations</li> </ul>
Gravity Gradient Field	Gravity Gradiometer	<ul style="list-style-type: none"> <li>• Stable platform required</li> <li>• Not yet established that moving-base gradiometer can be built to required accuracy</li> </ul>	<ul style="list-style-type: none"> <li>• Possible, if gradiometer development goals are met</li> </ul>
Magnetic Anomalies	Magnetometer	<ul style="list-style-type: none"> <li>• Self-field of vehicle</li> <li>• Magnetic temporal variations (including magnetic storms)</li> </ul>	<ul style="list-style-type: none"> <li>• Good, although performance can degrade severely during storms</li> <li>• Magnetometers may have to be towed to circumvent self-field effects</li> <li>• Could circumvent temporal variations using magnetic gradient measurements</li> </ul>

### 2.5.2 Limitations of Digital Data Bases

Provision of sufficient information in a digital data base for onboard navigation requires the capability to extract all of the necessary information from a wide range of source material. This information, when condensed into an onboard digital data base, inevitably suffers limitations imposed by the process of condensation. These limitations reveal themselves as sources of error in the navigation of the weapon system, as is discussed earlier in this chapter.

Figure 2.5-6 illustrates, for a correlation guidance system (TERCOM), the way in which such limitations affect estimated trajectories. A significant error source includes the measurement of the terrain altitude profile. The profile correlation process is also unavoidably affected by errors in the

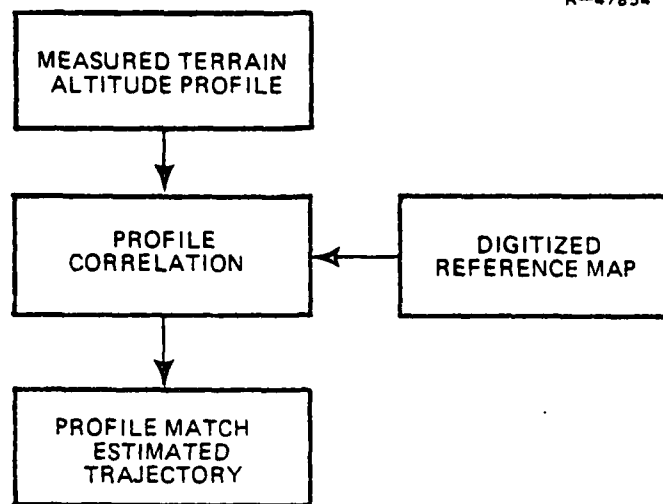
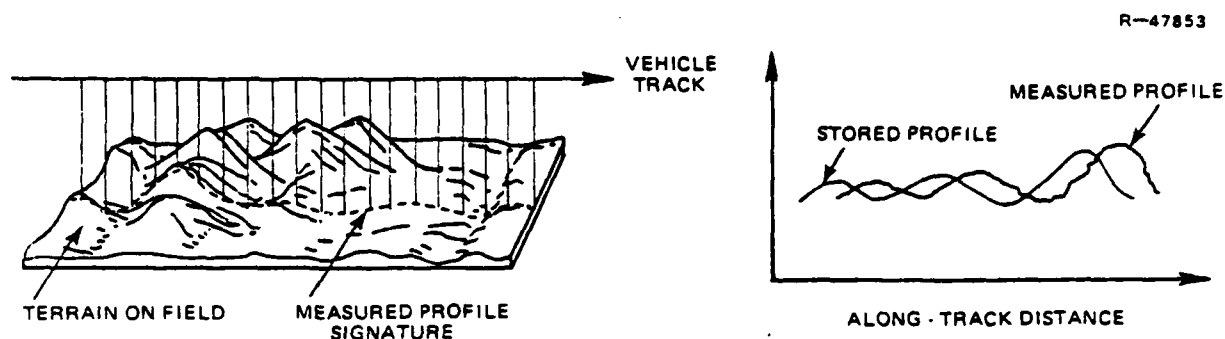


Figure 2.5-6 Real-Time TERCOM Computation

digitized reference map. As a result, profile correlation can never be perfect, and the determination of a true weapon system trajectory can never be absolutely achieved. The success of such correlation guidance concepts depends, then, on providing a sufficient level of accuracy to maintain navigation errors within the limits imposed by the mission objectives.

Figure 2.5-7 further illustrates the nature of the problem. The (arbitrary) vehicle track over a particular stretch of terrain develops an initial measured profile, in this case a vertical elevation profile. The processing problem for the onboard guidance system is then twofold. First, it must be able to extract, from its stored reference map, an equivalent stored profile based on an arbitrary trajectory across the reference map. Secondly, the system must correlate the stored profile with the measured profile. The correlation process must yield the precise location of the vehicle with respect to the local terrain as represented by the reference map.



- THE STORED PROFILE IS GENERATED FROM ONBOARD REFERENCE MAP USING INDICATED POSITION (INS)
- MAP MATCH OBTAINED BY SHIFTING THE STORED PROFILE BOTH ALONG AND CROSS-TRACK
- MATCH POINT IS DETERMINED BY MAXIMIZING CORRELATION BETWEEN STORED AND MEASURED PROFILES

Figure 2.5-7 Correlation Guidance (Signature Matching) Concept

Problem areas associated with a priori generation of a sufficiently accurate reference map arise from three principal sources:

- Limitations of source material and the source material collection process
- The translation process that condenses source data to an onboard reference map
- Variation in the appearance of the local terrain as it is sensed at mission time vs its appearance when the source material was collected.

The following discussion focuses on each of these three major error sources.

### Limitations of Source Materials

The materials used to develop onboard reference maps come from a wide variety of sources. The most important of these are photographic or other electromagnetic signature records taken as a result of reconnaissance activity. These materials unavoidably contain inaccuracies and errors.

All reconnaissance sensors, whether imaging or otherwise, will exhibit a fundamental maximum resolution. As a result, the best performance of the system is achieved when the sensor providing the source data has a much greater resolution than the sensors to be employed in the onboard mission. This resolution limitation does not fundamentally limit the mission performance. However, when source materials are obtained using available sensors, one cannot always guarantee that the source material resolution will be adequate for navigation system performance. Evaluation of source materials with respect to resolution, therefore, is a vital part of the generation of the digital data bases.

Another key element of the source limitation is the ability of the source material to provide metric accuracy. The process of using ground control and other systems to provide such accuracy for photographic-type products is discussed earlier in the chapter. For digital data bases derived from reconnaissance photography, metric accuracy will usually not be a major problem. However, for the newer types of source material (e.g., collection systems using side-looking radars or other sensors working outside the optical spectrum) metric accuracy may pose an important constraint on the maximum capabilities of the onboard system.

The taking geometry and the accuracy of stereo imagery source material also place limitations on accuracy. The taking geometry (i.e., the attitude and trajectory of the source collection sensor) necessarily will include masking or defilading effects in certain areas of the scene. For example, a building may hide the presence of a smaller building. When taking geometry can be optimized and when the required two views for stereo applications can be well controlled, these effects can be minimized for the important features of the scene. This may not always be possible, however, in practical situations. As a result, certain gaps or poorly-resolvable areas of the desired scene will result in corresponding errors in a derived digital data base representing that scene.

Temporal effects, including obsolescence of the source material, have a strong impact on reference map generation capability, in that it is desired that the map be an accurate representation of the scene as it will be viewed at mission time. Geological features of the earth's surface tend not to be subject to this limitation, but the so-called "cultural" features such as cities and town boundaries may change during the interval between the collection of the source material and its application in reference map form.

#### Limitations of the Translation Process

The translation of information contained in source materials by its condensation into a reference map suffers from certain fundamental limitations. The most important of these is that for a particular system of onboard guidance there is generally an optimum source data collection device. Unfortunately, the wide range of areas of interest and the practicalities of reconnaissance data collection prevent the exclusive use of "best" systems. Most of the time, source data



collection sensors are designed to provide the widest range and the greatest detail available. However, it is necessary to translate from the particular collection-optimized system to a navigation-optimized system. It is the latter for which the reference map is generated. The difficulties of translation include problems of stereo accuracy, surface material classification, and frequency-sensitive attributes of the sensed data.

Given that an adequate collection of source material is available, condensation to the practical size of a memory map for onboard application involves the process of editing. The available information must be gleaned to provide a sufficiently small but effective set of parameters for the onboard system. For example, a TERCOM stored map such as that shown in Fig. 2.5-8 contains on the order of 100-200 individual terrain-height values. The values are arranged in a matrix that covers several square miles on a side. The system is expected to use interpolation to arrive at intermediate values between the values in the matrix. A key issue is that of deciding how to position the regular grid for the reference map to pick up the truly significant terrain features in the local area. The process unavoidably involves editing some of the terrain data, leaving an incomplete set of data for the navigation system.

Another form of editing is indicated in Figure 2.5-9. In this case, the Pulse Doppler Map-Matching (PDMM) technique, a relatively simple reference map is produced by considering only a binary (high reflectivity vs low reflectivity) representation of the earth's surface. This extreme degree of editing, of course, results in great loss of detail in the reference map and imposes a requirement for more computational power in the correlation algorithm.

## STORED MAP

## MEASUREMENT

- Figure 2.5-8 Simplified Example of a TERCOM Fix



Related sources of translation limitation are the criteria for information selection in either automatic or human-guided processes. The completeness of the extracted information is also a limiting factor. Thus, the level of detail and the range of identifiable features limit accurate reconstruction of the precise location of the weapon system.

#### Mission Time Variations

At the time a correlation-guided weapon system actually performs its mission, a great many factors relating to the sensed scene come into play. Since these factors also affect the source material collection on which the onboard map is based, there is the possibility that a serious disparity will exist between the sensed scene and the reference map. Critical factors are:

- Illumination (especially in the optical domain) which may vary with sun angle or other illumination source geometry factors
- Cultural changes (i.e., new construction or changes in city boundaries)
- Mission-dependent viewing angle changes, velocity effects, and the effects of the specific weapon system trajectory.

All of these factors will influence the accuracy and recognizability of the scene as observed onboard the weapon system. Similarly, the onboard translation of the reference map into a representation of the observed scene is affected by the parameters of the mission trajectory. In addition, the time available to perform these calculations is affected by weapon system velocity. This implies that the onboard reference map should be generated as accurately as possible to present an easily correlated output. The fundamental limitations,

then, of the maximum resolution and detail which it is practical to put in an onboard reference map, are in conflict with the necessity to provide a great deal of fine-grained information to cover uncertainties at the time of the mission.

All of the foregoing considerations result in potential navigation inaccuracies that can only be compensated for by providing more extensive reference maps and more powerful onboard computation. Development in the future of a highly-accurate, long-term, stable source data base is a key priority in digital data base planning. Such a data base is being studied by DMA.

### 2.5.3 Operational Exploitation

The availability of extensive derived data bases in digital form plus additional source material (in the form of intelligence reports and reconnaissance imagery, especially in digital format) can be utilized efficiently for both mission planning and reference data base generation. Examples of planning activities involving selection of proper navigation routes and the identification of checkpoints and target scenes are presented in Chapter Six. The process of scene selection to generate reference maps for navigational updates and for target acquisition and final identification is the subject of this section.

The key to effective utilization of these digital data bases is the potential for interactive, cooperative processing involving both the human analyst and extensive computer support. The processing is called cooperative because both the man and the computer contribute unique but complementary capabilities. The analyst can, with training, exploit the

unique and (so far) unduplicated capabilities of the human psychovisual system to scan rapidly and select potential candidate scenes for a wide variety of purposes. The computer provides the capability to perform very detailed arithmetic functions on huge arrays of data in near-real time. The combination of the two provides the analyst with the range of enhancement techniques and other prompting aids to help direct attention to the most promising areas of a large scene. It supports provision of a "scratch pad" for rapid preparation of the necessary reports and recommendations, resulting eventually in reference map preparation.

The development of many of the techniques required to perform this cooperative processing, for operational exploitation, is still underway. Techniques for utilizing combined digital, textual, and image-format materials are still being explored and the optimum combination has not been achieved. However, excellent performance has already been obtained. The extensive experience gained, for example, by DMA in the Digital Land Mass System project, points the way to continued use of this mode of operation.

Note that the advent of new reconnaissance sensors will result in the requirement for human analysts to view "scenes" of image-like format, but representing the earth's surface as viewed at non-optical frequencies. In many cases, these images are readily understandable in their own right, with very little cueing required from other sources (such as maps). For example, images obtained from synthetic aperture radar, especially in urban areas, tend to look very much like photographic images of the same scene. On the other hand, images of the earth's surface taken in the infrared spectrum band tend to appear blurred (with respect to detail) and confusing because they reveal both reflected and emitted radiation.

Approaches to effective utilization of multispectral images of the same scene as a means of cueing the human analyst into identification of the appropriate scene content, for a given onboard sensor, are still being explored in the remote sensing community.

The key operational elements for reference map scene selection are:

- Scene content identification
- Reference effectiveness prediction
- Scene selection.

Scene content identification is, with present techniques, a highly subjective process performed almost entirely by human photointerpreters. The preparation for proper scene content identification, however, involves a great deal of photogrammetric processing and image enhancement, which is tailored towards sharpening the analyst's view of the scene. In cooperative man/machine processes for supporting scene content identification, a repertoire of pattern recognition and image enhancement algorithms must be provided so that each mission analyst can select the enhancement techniques most suitable to his or her personal approach.

The key items of scene content of importance to selection of navigation checkpoints and target acquisition and homing are:

- An accurately computed measurement grid to which the material available can be oriented
- Clear identification of surface material and classification cues

- Accurate identification of new construction and other changes in urban areas
- A method for designating the orientation and interrelation of specific surface materials and objects (e.g., buildings).

Reference effectiveness prediction consists of estimation of the received signal at the sensor and compensation for sensor imaging geometry. In the first case, consideration must be given to whether or not the sensor is active (i.e., whether the sensor illuminates the scene below or the illumination is provided by other sources, such as the sun). The complexity of such sensor-specific calculations is illustrated in Fig. 2.5-4 for an active sensor (synthetic aperture radar). Less complex active sensors, such as radar altimeters, present a more tractable estimation problem. The general problem for any particular class of sensors is to translate the spatial location, surface material classification, and orientation relative to the sensor into expected signal return contributions from all the elements of the digital data base. These expected returns must be integrated into a simulated sensor return waveform. The objective of this phase of reference prediction is to assess whether sufficiently unique information can be obtained from an area of interest in the digital data base to provide an accurate navigational update, as described in Section 2.3.7.

Compensation for sensor imaging geometry, which in a general sense includes sensor platform velocity effects, is a parallel effort in the reference effectiveness prediction process. It is inherently part of sensor return estimation. Note, for example, that the Doppler effect (frequency change) due to vehicle motion during the illumination interval, and the changing relative orientation of the sensor's receiving antenna (or

lens) to the emitters/reflectors making up the scene, must be taken into account during the computation of the expected contributions from each source.

Another aspect of the sensor-imaging geometry problem is that the reference map provided to an onboard system possesses a standard orientation relative to the earth's surface. The sensor on a moving platform may approach the mapped area at a different relative orientation. The onboard system must provide a means for comparing the sensed image with the reference map in a common orientation and coordinate frame. As a result, onboard processing must warp and rotate the sensed image to match the orientation of the reference map. In this process the actual onboard signals received for particular areas of a sensed image are recombined in complicated ways. This subject is discussed in Unit Three under Imaging Operations. Its effect on reference prediction is to lower the prediction fidelity of sensor returns.

The final scene selection process involves an iterative application of reference predictions for varying flight trajectories over the mission area. Received estimates are evaluated for the availability of sufficiently detectable signatures to provide navigational accuracy. The uniqueness of those signatures (to avoid false positioning) is also taken into account. It is evident that such a process requires extensive computational resources and interactive communication between the mission planner and the supporting computers. The net result of the process, however, is the selection, from the available digital data bases for the general area, of specific, highly effective data subsets. These subsets can be formatted for use as onboard reference maps in both the navigational update and target acquisition/final location phases.



#### 2.5.4 Masking and Profiling

Digital data bases of terrain elevation provide the means for computer support of two critical operations in the preparation of a mission flight plan. These are masking and profiling. Masking is the process of determining, for a postulated flight trajectory, whether or not defensive installations such as radar will be able to observe the weapon system en route. The operations involved are conceptually quite simple, in that what is required is to determine whether there is a clear line of sight (LOS) between known or suspected defensive installations and all points within a reasonable range along a flight trajectory. Given a table of terrain heights or other equivalent representation of the area in question, several approaches may be applied to perform the calculations required. For example, these may be based on simple projection of the LOS from the defensive position to the elevation of the trajectory point. Another technique is checking each cell of an array of elevation data to determine whether or not the elevation of the terrain at that point will block the signals emitted by the radar (or other defensive device).

It is apparent, however, that these relatively simple calculations must be performed many times for a given flight plan and that computer support is therefore an absolute requirement. On the other hand, the general problem of selecting a mission trajectory for testing is not a completely computer-solvable task. Given the other constraints on the mission flight trajectories, such as those discussed in the previous section, it is apparent that tentative selection of mission trajectories will require interactive and cooperative computer support for the mission planner. In providing this kind of support, it is absolutely essential that highly capable and

sophisticated graphics displays be used to simplify the interaction between the mission planner and the computations, and thereby shorten the time required for determination of candidate routes for the weapon system.

Once a collection of acceptable routes for approaching and acquiring the target is determined for a particular mission, and a collection of navigation update scenes and scenes of the target area is obtained, then the final selection of the optimum and the best alternative routes can be made. This selection is based entirely on the characteristics of the weapon system's onboard sensors and navigation equipment.

For those weapon systems that depend on identification and tracking of a desired terrain profile (e.g., TERCOM) the process of profiling is required. The objective of this process is to determine the most distinctive and least ambiguous approach profile to maximize either the position update accuracy or the accuracy of the target acquisition process. The key issue is that, for a given stretch of terrain, it is likely that a great many profiles corresponding to the ground trace of the weapon trajectory will be similar and therefore ambiguous. The similarity referred to, of course, is similarity as observed by the onboard computation system, particularly if it involves some form of correlation process.

Essentially, the profiling process is one of pattern recognition and feature extraction. The features (i.e., the significant characteristics of the profile as measured by the onboard correlation process representing those same profiles) can be plotted in a so-called feature space representation. Figure 2.5-10 illustrates such a representation. In this figure it is indicated that profiles representative of three separate trajectories (S1, S2, and S3) are under consideration.

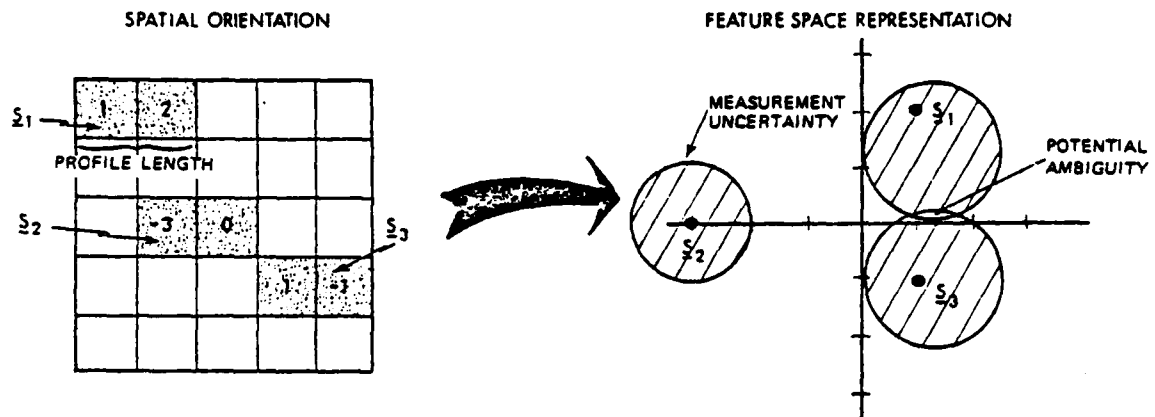


Figure 2.5-10 Profile Representation in Feature Space

The feature-space dimensions are the values obtained by the correlation process for each of the significant measures of profile characteristics. Profiles well separated in feature space are readily distinguished by the correlation processor and can, therefore, be used unambiguously to guide the weapon system.

A scene for which all profiles tend to cluster in feature space represents a difficult problem for the weapon system mission planner; if the scene represents the target area then the probability of kill is likely to be quite low. However, there is often a possible selection of unambiguously detectable profiles. The process of profiling requires that there be an interactive mode of candidate profile selection available to the mission planner.

An example of the application of scene ambiguity measures to feature space analysis is given in Fig. 2.5-11. In this case, a general process is to determine the probability of ambiguous correlation for the given trajectories over a range of available scenes. The algorithm for determining this probability may be qualitatively described as follows:

- Search for minimum distance between possible profiles within the acquisition area
- Ignore profiles that are spatially close to each other (i.e., those for which ambiguity would not seriously affect navigational accuracy)
- Use minimum distance in feature space and measurement error statistics to compute probability of ambiguous correlation for the given profile.

#### SAMPLE RESULTS

NEW ENGLAND TERRAIN COR- RELATION TEST Scene No.	MINIMUM DISTANCE IN FEATURE SPACE (ft)	No. of OPERATIONS REQUIRED*
1	19.7	630,000
6	28.4	440,000
3	55.6	710,000

\* 100 Trial Monte Carlo for some Search Region (30x30 cells)  
Requires ~ 5,000,000 Operations

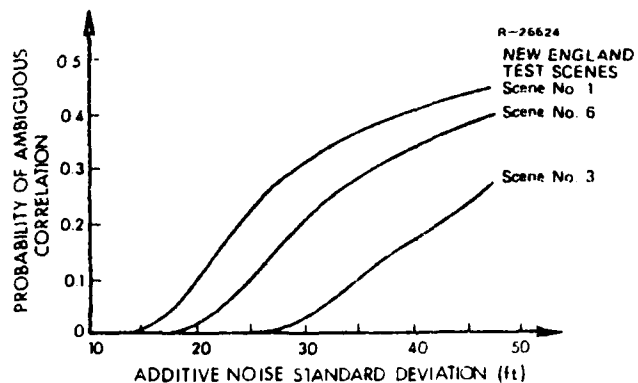


Figure 2.5-11 Feature Space Scene Ambiguity Measure

The technique is fundamentally a worst-case analysis of scene ambiguity. For selection scenes in a New England area, Fig. 2.5-11 is illustrative. Sample results indicate quite clearly that Scene No. 3 is superior to Scenes No.1 and No. 6 in providing a high probability of unambiguous correlation. This holds true even with significant amounts of additive noise in the measurements. However, it is important to note in the sample results that a very large number of computational operations are required to estimate the degree of ambiguity in each scene.

#### 2.5.5 Graphic Display

The method of displaying the combined effect of manual decisions and computer-based data retrieval operations (for purposes of effective man/machine cooperation in profiling and targeting) is a very important aspect of the digital data base utilization problem. It is essential that the display system fully utilize the human operator's ability to recognize important objects and to combine many items of information. This ability is most readily tapped by presenting information to the operator in visual form. At the same time, an effective display must be able to utilize the computer's ability to perform a great many detailed calculations in a short time. That is, the display mechanism must be well tailored to presenting the results of these calculations in a parallel and overlapping manner, so that such information as terrain elevation can be rapidly absorbed by the human operator along with the results of target-oriented feature recognition algorithms. Another aspect of the problem is that the display mechanism chosen should also support ready access to a wide variety of data bases. Of course, such data must be digitally encoded and include information based on maps, previous intelligence reports, and the like.

The principal problem is one of narrowing the range of alternatives for weapon system trajectories, selecting aim points, and choosing target approaches. The process of narrowing the range of alternatives must be accomplished in accordance with a combined set of constraints. It is apparent that a great many approach paths are possible for any given weapon system/targeting combination. At the same time, a great many reasonable altitudes could be utilized with more or less exposure to observation by defensive forces, as is discussed in previous sections. It is essential, therefore, to present, in concise form, the location of likely paths that connect useful navigation system update points and targeting locations. This must be done in such a way that the human operator can rapidly identify the options available to him and select specific paths by interacting with the graphic display terminal. Typically, the result of these selections is obtained by iterating; that is, given a set of reasonable criteria, the supporting computer system will present the range of alternatives on a graphic display. The operator will select one of these alternatives and then request further, more detailed analysis of survival probability, etc. The result of such iteration should be to rapidly develop specific mission plans for any combination of weapon system and target characteristics.

The graphic displays chosen for such applications should have at least the following characteristics:

- Color graphics capability must be provided to enable the human operator to grasp the complexities of overlapping constraints
- An interactive command/response capability must be provided to enable the human operator to perform iterative or repetitive analyses in the process of route and target selection

- The system should be able to support imaging capability in conjunction with overlapping graphics and color selection so that the full range of available intelligence information can be presented to the operator.

An example of a workstation combining high resolution, graphics, imaging, and color capabilities is shown in Fig. 2.5-12. The system shown in the figure incorporates a modern minicomputer-based multi-workstation concept. Each of the workstations includes a variety of modes of display of information to the operator. Some of these are purely textual. The more useful ones provide for both keyboard and trackball control (or other means of analog, interactive operator control). Color displays are also considered to be essential. Information can be presented to the operator in several important ways with such equipment:

- Purely textual displays, which can be amplified and made more comprehensive in response to operator queries
- High capability color graphics, which can be used to display highly complicated scenes including artificially-generated three-dimensional views of a given area
- Independently selectable color coding can be provided to allow depiction of selected areas color coded according to the constraints that they meet
- Color merging capability (inherent in most color displays), such that areas meeting several constraints are shown in colors resulting from the combination of colors representing individual constraints
- Free selection of viewing mode for the operator, provided by a comprehensive capability for zooming and scrolling.

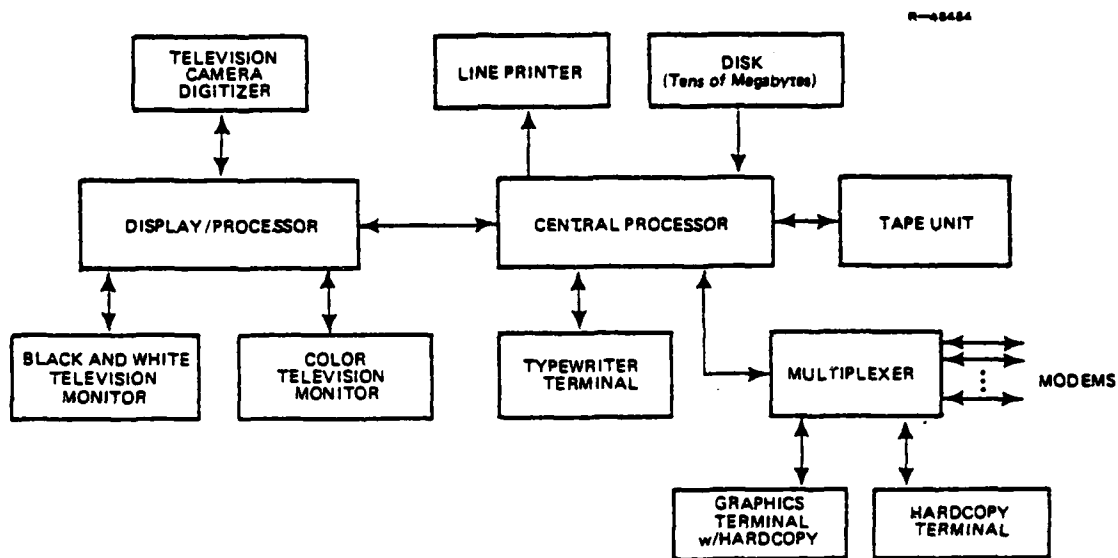


Figure 2.5-12 Image Processing Workstation

Figure 2.5-13 is an artist's reproduction of a photograph taken from an imaging/color graphics display such as illustrated in Fig. 2.5-12. Indicated in Fig. 2.5-13 are some of the inherent capabilities of such a display. In particular, it shows how computer graphics can be overlaid on a three-dimensional view produced from an elevation matrix. An important part of the color overlay (shown in Fig. 2.5-13 as different shading patterns) is the "flooding" portion which demonstrates how a specific constraint is applied to the area in question. In this case, all of the area in the field of view at or below a given elevation is colored (shaded) with a specific tone, to show the operator the range of paths across this terrain that might be followed by a weapon system staying below that elevation limit. The limitations of reproduction in this case prevent clear exposition of the ability to show areas satisfying combined constraints. However, it requires





Figure 2.5-13 Graphics Display Example

very little stretch of the imagination to conceive of corridors being displayed in very distinctive colors (shades) to correspond to areas below a given elevation and simultaneously masked by terrain from all known defensive positions in the area. Further complications can be added almost without limit so that the human observer can rapidly select the possible paths a weapon system might take in traversing the given terrain. Such capabilities are of the utmost importance in being able to provide rapid and responsive targeting and mission trajectory determination.

## CHAPTER SIX

### APPLICATIONS TO MISSION PLANNING

#### 2.6.1 Route Analysis

A growing number of military mission scenarios requiring accurate weapon delivery in enemy-controlled areas involve low-altitude penetration vehicles (e.g., cruise missiles and manned bombers). The key requirements for such a weapon delivery system include precision navigation and terrain-following/obstacle avoidance systems, which increase vehicle survivability by permitting lower altitude penetration and lower visibility through terrain masking. The navigation systems of such vehicles use aided-inertial navigation techniques such as terrain-following radar to provide master inputs in the vertical plane. In addition these sensors provide information for route determination in the horizontal plane. Especially at low heights, the principal problem is one of continuous dynamic control in both horizontal and vertical planes relative to the terrain.

For cruise missiles and other low-altitude delivery systems, the important performance criteria are: 1) probability of detection and neutralization by the enemy; 2) range loss; and 3) probability of crashing (for which the term clobber is widely used). These factors are all interdependent, in that altering one will most likely cause the others to be altered too.

There is an obvious dilemma for a long-range cruise missile flying over land masses. It is desirable to reduce the visibility of the missile to defense radar by flying at a low clearance altitude, with the minimum altitude over flat

land being limited by such obstacles as power cables, television antennas, tall buildings, etc. Simultaneously the system must achieve an acceptably small probability of crashing into the ground (clobber probability). In order to maintain a low clearance altitude, the missile must follow the vertical contours of the terrain. These terrain-following maneuvers, however, also impose a restriction on mission planning; they increase the total aerodynamic drag and, consequently, the rate of fuel consumption. This increased fuel consumption decreases the maximum range of the missile for a given fuel capacity. In short, range loss is related to the amount of vertical acceleration. Choosing a route to target that minimizes the changes in vertical acceleration also maximizes the range. In addition, it is preferable that a route have terrain contours without high spatial frequencies -- i.e., the terrain should not be too rugged or precipitous for the terrain-following radar system and the vehicle maneuvering abilities to detect and safely avoid.

The possibility of encountering enemy defense systems along such a route must also be considered. Therefore, another factor in route analysis is terrain masking. Here, a route is chosen that takes advantage of the local terrain to block the target's or other defensive station's view of the incoming vehicle for as long as possible. This route must still be consistent with the requirement of minimizing clobber.

Route analysis also involves an understanding of the types of enemy defenses, and especially defensive sensors, expected between the vehicle launch site and the target area. As mentioned above, the terrain contours should be used as much as possible to avoid being detected. However, where detection is deemed likely, it may be advisable to include listening devices and electronic countermeasures on board the

vehicle. The kinds of listening devices and countermeasures available for a given missile or aircraft may make one potential route more attractive than another simply because of the nature of the enemy defense mechanisms known to be deployed along the various routes.

An option for advanced cruise missiles involves multiple routes. In this case the missile makes an autonomous decision at some point along its initial route to the target either to continue onward or to pursue some other route. The basis for this decision may be the detection of unexpected enemy defensive systems. Another scenario involves missile arrival in the vicinity of the target but failure to detect the target -- if, for example, the target has been destroyed by a previous missile. The mission plan may allow the missile to abandon the primary target and pursue a new course to another target. Such route changes are, of course, still dependent on an analysis of the available range left for the missile at the decision point. In situations where the time of arrival of a missile at the target is of crucial importance (for example, in the simultaneous delivery of many warheads), there is less flexibility for choosing a new course. In addition, a cruise missile diverted automatically to a new, second-priority target may be destroyed enroute by blast effects from other cruise missiles that have already reached their targets. This phenomenon is called fratricide.

#### 2.6.2 Clobber Analysis<sup>(†)</sup>

Terrain following techniques allow the vehicle to maintain a low clearance altitude by following the vertical contours of the terrain. This can be done by the use of a

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(†)This section contains material at a more advanced level than the rest of the text.

forward-looking radar sensor. However, in addition to added costs, these systems radiate energy forward, compromising the low visibility which is achieved by the small clearance altitude, and thereby increasing the probability of detection. There are some forward-looking radar systems and techniques -- such as spread spectrum transmission and coherent laser scanners -- which are expensive and complex, but possibly satisfactory as a forward-looking sensor. Nevertheless, for a cruise missile, it is generally desirable to do terrain-following without the use of active forward-looking devices -- that is, to rely on downward-looking radar.

Terrain may be thought of in terms of local high-frequency excursions about a low frequency, possibly non-stationary, trend. This is illustrated in Fig. 2.6-1. Analysis has shown that the trend appears to have frequency content less than 0.03 cycles/kilometer. In Fig. 2.6-1,  $h$  is the terrain elevation above some reference ---for example, sea level. This is decomposed into a terrain trend,  $h_{\text{trend}}$ , measured from the reference, and an excursion,  $T_{\text{ex}}$ , about the trend.  $A$  is the altitude of the missile trajectory above the reference and  $A_{\text{CL}}$  is the clearance height of the missile trajectory above the terrain as measured by a radar altimeter. Note that

$$A_{\text{CL}} = A - h.$$

Because of the frequency separation between the terrain trend and the relatively high-frequency excursions about the trend, the terrain model can be simplified for control system design by considering only the high-frequency excursions. This approach assumes that the missile has sufficient maneuverability to follow the high-frequency excursions and also that the range-loss penalty imposed by following the terrain trend is negligible.

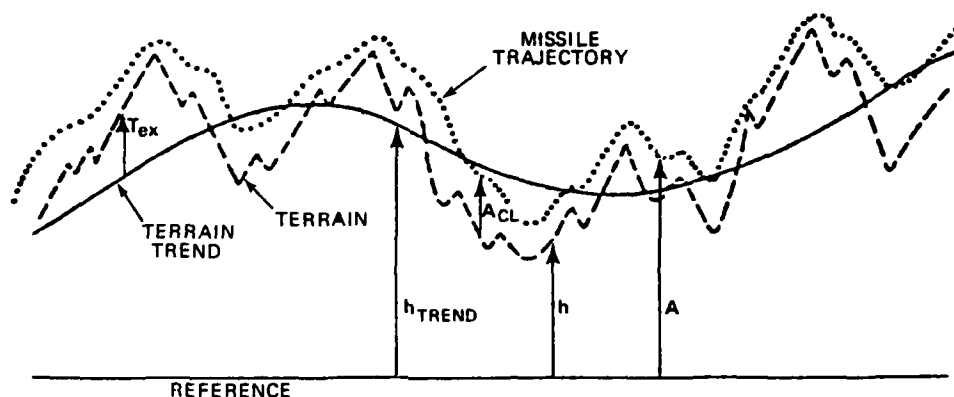


Figure 2.6-1 Terrain and Missile Trajectory

A statistical model for the high-frequency terrain components is required for analytical study of the terrain-following problem. These fluctuations may be regarded as a realization of a stationary Gaussian random process.

One such model is given by

$$\phi_x(k) = \frac{2\sigma_T^2 k_T^2}{k^2 + k_T^2} \quad (2.6-1)$$

where

$\phi_x(k)$  = power spectral density (PSD) of  $T_{ex}$

$k_T$  = frequency of the terrain in cycles/km

$\sigma_T^2$  = variance of the terrain in  $m^2 = E[T_{ex}]^{2*}$

$k$  = Fourier spatial frequency variable having units of cycles/km

\*E is the Expectation Operator of probability theory.

This relationship can be converted to time units by assuming a velocity of the missile ( $V$  in m/sec) over the terrain. Then the frequency variable,  $k$ , and the terrain cutoff frequency,  $k_T$ , (as seen by the control system) become

$$\omega = \frac{2\pi V k}{1000} \quad (2.6-2)$$

$$\omega_T = \frac{2\pi V k_T}{1000} \quad (2.6-3)$$

and the PSD of the terrain fluctuations becomes

$$\phi_T(\omega) = \frac{2\sigma_T^2 \omega_T^2}{\omega_T^2 - \omega^2} \quad (2.6-4)$$

where  $\omega$  and  $\omega_T$  have units of radians/sec.

A terrain signal with the above PSD can be generated by passing white noise through a linear first-order filter with frequency response (shaping) given by

$$F(\omega) = \frac{1}{\omega + \omega_T} \quad (2.6-5)$$

In the missile, an error signal  $E$  is formed as the difference between the above-derived terrain signal and  $A$ . (In actuality, the missile will fly at some offset altitude, or average clearance altitude,  $A_{CL}$ .) This determines the vertical acceleration command to be implemented by the missile's guidance system.

The purpose of terrain following is to reduce the visibility of the vehicle to defense radars and thus reduce the probability of intercept. The reduction in hit probability is achieved at the expense of increased probability of

impacting the ground (clobber probability). Thus, there is a requirement to fly as low as possible and still maintain a reasonably small probability of clobber. The control system which results in the lowest probability of clobber for a given average clearance will enable the missile to fly at the lowest possible average clearance altitude to achieve a specified probability of clobber. Thus, the objective is to design a control system which will minimize the probability of clobber while satisfying the other constraints of the problem.

There are a number of ways to compute the probability of clobber. One method results in the expression

$$P_{\text{clobber}}(1) = 1 - e^{-ft} \quad (2.6-6)$$

where

$t$  = flight time (sec)

$f$  = mean frequency of clobber (clobbers/sec) as defined below:

$$f = \frac{1}{2\pi} \frac{\sigma_{\dot{A}_{CL}}}{\sigma_{A_{CL}}} \exp \left[ - \frac{A_M^2}{2\sigma_{A_{CL}}^2} \right] \quad (2.6-7)$$

where

$\sigma_{A_{CL}}$  = standard deviation of error in achieved clearance altitude  $A_{CL}$

$A_M$  = mean flight altitude

$\sigma_{\dot{A}_{CL}}$  = standard deviation of the clearance rate (error rate)

A second formulation gives this expression for the probability of clobber:



$$P_{\text{clobber}}(2) = \frac{1}{2} - \text{erf}(A_M/\sigma_{\text{ACL}}) \quad (2.6-8)$$

Note that  $\text{erf}(x)$  is the error function, defined as

$$\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp(-x^2) dx \quad (2.6-9)$$

The results of each method are dependent on the initial scenario, including desired flight characteristics and expected terrain features. The important thing to note is that both of the expressions show that the clobber probability for a given average clearance altitude varies with  $\sigma_{\text{ACL}}$ . Thus the control system which results in the smallest  $\sigma_{\text{ACL}}$  while still satisfying the constraints on the missile dynamics, radar altitude sensor resolution, etc., will achieve the smallest probability of clobber.

UNIT TWO  
REVIEW EXERCISES

Chapter Three

1. Must the integrations illustrated in Fig. 2.3-1 be mechanized in an inertial reference frame? Discuss.
2. Why is it not possible to use an inertial system's accelerometers to measure and subtract away the effect of gravity?
3. A locally-level mechanized inertial system provides navigation information for an aircraft flying at constant altitude along a meridian at 800 km/hr. Neglecting earth rotation, what is the angular slew rate of the inertial platform with respect to inertial space? (Hint: Consider the angle between the platform-indicated vertical and the earth's spin axis.)
4. Consider a vehicle with a local-level-mechanized inertial system which is stationary at the equator, at a place where each component of the deflection of the vertical is  $10 \text{ sec}$  and the vertical gravity disturbance is 50 mgal. Suppose that the position error is one km in the two level channels and 3.05 m in the vertical. Also suppose that the gyro errors have results in  $\psi$  angle errors of  $5 \text{ sec}$  in all three channels. The accelerometer errors are  $5 \times 10^{-6} g$  for each accelerometer (where the  $g$  is a unit of acceleration equal to nominal sea-level gravity --  $9.8 \text{ m/sec}^2$ ). Tabulate the value of each term in Eqs. 2.3-11 through 2.3-12. Note that these terms correspond to the signals

entering the acceleration summing nodes in Fig. 2.3-9. Use units of  $10^{-6}$  g. What does this example illustrate about the cross-coupling terms of Fig. 2.3-9?

5. Suppose an inertial system uses a  $GM/r^2$  gravity model instead of a reference ellipsoid gravity model. If the nominal surface value of gravity is 978.049 gal, use the approximate ellipsoidal gravity formula

$$g = 978.049(1 + 0.0053 \sin^2 \phi) \text{ gal}$$

to compute the vertical acceleration error sensed by the system at a latitude of 45 deg. At this rate how long does it take for 100 m of altitude error to accumulate? Assume that the vertical gravity disturbance is zero at the location of the system.

6. Suppose a guidance system is to be compensated for gravity by using a model of the form of Eq. 2.3-20. What error sources might be associated with such a model if the maximum value of  $n$  is 20.

(Section 2.3.5)

7. In a ballistic missile application it is desired to erect the inertial guidance platform with the accelerometer sensing axes symmetrically distributed about the local vertical (gravity vector) and with one accelerometer sensing axis pointing downrange. Assuming the use of a mutually orthogonal set of accelerometers, what are the nominal (error-free) accelerometer outputs ( $f_x$ ,  $f_y$ , and  $f_z$ ) when alignment is complete? (Hint: Use Eq. 2.3-23 and symmetry considerations.)

8. A local level inertial platform is aligned in a stationary vehicle with the (orthogonal) x and y sensor axes in the level plane at a point on the earth's surface at which the geodetic and astronomic verticals coincide (zero deflections of the vertical) and the common latitude is  $\phi$  deg. The x gyro axis points 5 deg north of east. The gyro torquing rate commands from the navigation computer are given by:

$$\begin{aligned}\omega_x &= 0 \\ \omega_y &= \Omega \cos \phi \\ \omega_z &= \Omega \sin \phi\end{aligned}$$

where  $\Omega$  = Earth Rotation Rate relative to Inertial Space. What are the observed platform tilt rates ( $\dot{\phi}_x$  and  $\dot{\phi}_y$ ) about the platform x and y axes respectively? (Hint: Compute the components of earth rate appearing about the actual (misaligned) sensor axes ( $\Omega_{xH}$  and  $\Omega_{yM}$ ) and use Eq. 2.3-27 to get tilt rates.)

9. Using the small-angle approximations,  $\sin 5^\circ \approx 0.1$  and  $\cos 5^\circ \approx 1.0$ , evaluate the tilt rates  $\dot{\phi}_x$  and  $\dot{\phi}_y$  derived in Problem 8 for vehicle latitudes  $\phi_1 = 30$  deg N and  $\phi_2 = 70$  deg N. Answer the following questions:
- (1) Which accelerometer senses a tilt rate that can be used to estimate and/or control the platform azimuth misalignment?
  - (2) What is the change in the magnitude of the gyrocompassing control signal as the alignment latitude increases?

- (3) What does this difference tell you about inertial platform self-alignment capabilities at high latitudes in the presence of accelerometer time-varying errors or gyro bias drift errors?

10. An error-free, north-slaved, local level inertial platform is self-aligned in a stationary vehicle at a point on the earth's surface at latitude 30 deg N. An easterly deflection of the vertical,  $\eta = 10 \text{ } \overline{\text{sec}}$ , exists at the alignment location. The existence of this deflection of the vertical is not accounted for in the navigation computer. Evaluate the effect of this omission on:

- (1) Platform tilt errors
- (2) Platform azimuth error

relative to the (geodetic) navigation coordinate frame when alignment is complete. Repeat the evaluation when the vehicle latitude is 70 deg N and the same conditions exist.

11. An aircraft is equipped with an avionics suite that includes a gimbaled inertial platform, a Doppler radar set and a LORAN receiver. Two distinct and separate modes of in-flight initialization are mechanized in the system:

- (1) Doppler (Velocity) - Inertial
- (2) LORAN (Position) - Inertial.

In the former mechanization the Doppler velocity inputs are transformed directly into inertial sensor coordinates,

using resolvers mounted between the gimbals of the inertial platform, prior to comparison with the inertial system velocity outputs. In the latter mechanization the LORAN receiver outputs are converted to geodetic latitude and longitude in the navigation computer and compared with the latitude and longitude outputs of the inertial system to form alignment control signals.

- (a) What is the basic difference in the dynamics of the inertial platform alignment process between the two mechanizations?
- (b) Can a complete inertial system initialization be achieved if the LORAN receiver is inoperative?

(Section 2.3.6)

12. It is 1985! USAF is considering the deployment of a new ballistic missile system as an augmentation to the aging Minuteman force. In order to obstruct a surprise attack by opposing forces, this new missile will be deployed in truck-like transports which will roam at random through the deserts of southwestern CONUS. The missiles will be erected and fired from the moving transporter when the order to launch is received.

A weapon system CEP of 75 m is desired, and an allowance for the guidance system contribution to this total has been set at 30 m, including the effects of launch site uncertainties. The latest generation of inertial guidance hardware indicates that there will be no problem in meeting this accuracy goal if the initial condition errors can be controlled. The transporter has accordingly been equipped with special radio devices that allow it

to determine its position continuously to an accuracy of about 3 m. Has the initial condition problem been fully addressed? Discuss.

(Section 2.3.7)

13. Why are aided inertial navigation techniques preferred over unaided inertial navigation techniques? Give examples of the two types of Aided INS errors, and how they are counteracted.
14. What is the fundamental technique behind terrain-matching navigational methods? Give examples.
15. Autonomous terminal-homing guidance systems generally use scene-matching concepts to determine the location of the vehicle or its position relative to the target. Give three specific, different difficulties that can be encountered in preparing and using the reference imagery model by the vehicle's onboard navigation system for correlation guidance.
16. Suppose that a local-level mechanized inertial platform points toward a star located at the zenith. Discuss the improvement in system azimuth error that can be gained from this measurement.
17. Consider an inertial system which has no platform misalignment (i.e.,  $\theta = 0$  in Fig. 2.3-24). Can any error-free star fix correct all other system errors? Discuss.

Chapter Four

18. How can Point Positioning Data Bases (PPDBs) be used to support tactical bombing operations?

## Chapter 5

19. What are the key considerations for selection of a navigation/ guidance concept based on an on-board reference map?
20. What are the principal sources of navigation errors which arise from the use of an on-board reference map?
21. (a) What are the principal tasks to be accomplished in preparing reference maps for specific missions?  
  
(b) What implications do these have for the tactical or strategic mission planner?
22. What are the elements of an observable feature in a terrain data base?

## Chapter Six

23. Give the three important interdependent performance criteria, as related to route analysis for low-altitude, high-speed weapon delivery vehicles such as cruise missiles penetrating enemy-controlled areas.
24. Refer to Eqs. 2.6-5 and 2.6-6. Assume that a particular cruise missile flies with a standard deviation of error in achieved clearance altitude ( $\sigma_{ACL}$ ) of 25 ft, and a standard deviation of the clearance rate ( $\sigma_{\dot{ACL}}$ ) of 5 ft/sec. What is the lowest mean flight altitude that can be achieved, for a 2-hour flight, that results in at most a 1% probability of clobber?



UNIT TWO  
READING LIST

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